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Production, Time and Value:
Accounting Capital in Horizontal Innovation Models

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1. Prologue

In traditional aggregate (or one-sector) neoclassical growth models (e.g. Solow, 1956), the economy is pictured as one where the final good is produced by means of labour and capital goods and where capital goods are in turn 'produced' directly by 'foregoing' the consumption of the final good. The first production process is construed as involving some lapse of time, whilst the second 'production' process is not: the transformation of the final good into capital goods is immediate. If we call the process of production which is completed in a positive length of time a 'layer' of production, the economy of traditional neoclassical models is a 'single-layered' economy.[1] A main characteristic of a 'single-layered' economy is that no problems arise regarding 'value' and the measurement of 'capital'. The 'quantity of capital'—which is an aggregate of capital goods used in production and also on which interest accrues (thus, which is measured in value terms)—is simply the quantity of the foregone final good.

This way of envisaging the economy is in contrast with the way that economists in the tradition of Austrian capital theory such as Menger, Böhm-Bawerk, Wicksell and Hicks used: here, the economic process is viewed as starting from the application of the original factors of production (labour and land) and, through a series of intermediate stages, leading to the production of the final good. Each stage of production takes time, and the stages are sequentially connected. Thus, each stage of production constitutes a 'layer' of production. The Austrian economy is a 'multi-layered' economy.

It is easily seen that the economy depicted in recent one-sector endogenous growth models, such as the AK model and the knowledge-spillover models (for example, see Barro and Sala-i-Martin, 2004, Ch. 4), is a 'single-layered' economy. By contrast, the economy in horizontal innovation models in endogenous growth theory is, at first sight, very similar to the Austrian one.[2] The economy consists of a *multiple* number of 'sectors' which are sequentially connected. Typically there are three sectors: the R&D sector, the intermediate goods sector, and the final good sector. The R&D sector produces new designs; the intermediate goods sector uses these new designs (plus the

[1] The term 'layer' conveys two senses: positive thickness (however thin) and sequence.
[2] It is no coincidence that the title and the subtitle of the present paper, which deals with horizontal innovation models, contain three key words which appeared in John Hicks's trilogy in the tradition of Austrian capital theory—'value', 'capital' and 'time'.

stock of old designs) to produce intermediate goods; the final good sector in turn uses intermediate goods to produce the final good; finally, the final good is either used up as consumption or ploughed back as investment. Production in these 'sectors' is sequentially connected. This economy is similar to the Austrian one and thus *in principle* ought to be a 'multi-layered' one.[3]

However, as the following pages will claim, the similarity of (currently available) horizontal innovation models with the Austrian formulation is a mirage. The truth is that, by a sleight of hand, these models reduce the essentially 'multi-layered' economy to a 'single-layered' one.[4] Production takes time only in one of the sectors, whilst in the remaining sectors production is completed in a timeless setting. The result is an economy with a single layer. Thus, as with the traditional neoclassical 'single-layered' economy, no problems arise in connection with value or the measurement of the 'quantity of capital'. This is the result of high dexterity in modelling—but, in our judgement, at the cost of economic reality and logic.[5]

Section 2 begins our argument by analysing the 'single-layeredness' of the economy considered in the traditional aggregate neoclassical growth models. Section 3 dissects a representative model of horizontal innovation—Barro and Sala-i-Martin's (2004, Ch. 5) 'nondurable lab-equipment' model—in order to see how an essentially 'multi-layered' economy is reduced to a 'single-layered' one, with time taken away from all but one sector (and that in internal inconsistency). Section 4 reinstitutes time in production, thereby recovering the 'multi-layeredness' of the kind of economy considered in horizontal innovation models. Section 5 compares the result of the time-recovered economy with that of the time-removed one. Section 6 concludes.

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[3] The representative models in the horizontal innovation literature are Romer (1990), Rivera-Batiz and Romer (1991) and Bénassy (1998), each representing the three groups of models ('baseline', 'lab equipment', and 'labour for intermediates' models). See Gancia and Zilibotti (2005) for a useful survey and Barro and Sala-i-Martin (2004, Ch. 6) for a good exposition.

[4] The starting point of the horizontal innovation literature is Romer (1990). It is suggestive that its predecessor, Romer (1987), explicitly deals with a 'single-layered' economy: output is 'allocated between consumption … and investment in additional capital,' and 'foregone output … [is] converted one-for-one into new capital' (1987, p. 60).

[5] The argument below will equally apply to 'Schumpeterian models of quality ladders'. We shall however conduct argument in reference exclusively to horizontal models. The Schumpeterian models are represented by Aghion and Howitt, (1992); Barro and Sala-i-Martin (2004, Ch. 7) provides a good exposition.

2. The reign of time: a 'single-layered' economy

The production process of a traditional neoclassical growth model (e.g. Solow, 1956) can be schematized as in Figure 1.

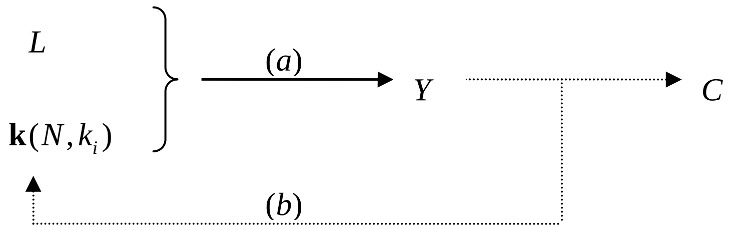

Figure 1: The production processes in the traditional model

Y = final output;
C = consumption;
L = labour;
$\mathbf{k}(N,k_i) = (k_1, k_2, ..., k_n)$ = the N-vector of quantities of N capital goods.

The final good is produced using labour and 'capital' (Process *a*). The produced final good is either used up for consumption or 'foregone' for investment. The 'production' of capital goods, which are to be used in the production of the final good, requires no input other than the foregone final good (Process *b*).

The 'production' of capital goods is *immediate*; that is, there is no elapsing of time between the application of the final good as the input and the 'production' of capital goods as the output—all that is required is abstaining from (= foregoing) consumption. Supoose the *i*th capital good requires ζ_i units of the final good for unit production.[6] That is, Process *b* is represented by the following methods of production:

[6] We are here referring to *different* kinds of capital goods, without questioning *how* the homogenous final good, as the sole input in production, is transformed into these 'different' kinds of goods. The more usual construal is that capital goods are homogenous to each other and also to the final good, so that $\zeta_i = 1, \forall i$. We use the setting of (pseudo-)multiple capital goods in anticipation of horizontal innovation models, in which the homogenous final good is transformed into different intermediate goods depending on different 'designs'. In horizontal innovation models, where the

(1) ζ_i units of the final good → 1 unit of the *i*th capital good

Here ζ_i in itself stands for the physical quantity of the final good foregone as the input in producing one unit of a capital good. Conceptually this quantity *must* be differentiated from the unit *value* (or price) of the *i*th capital good. The determination of the value of a good involves (*i*) the choice of the standard of value and, more importantly for our current argument, (*ii*) the consideration of the period of time during which inputs are locked up in the process of production. This lock-up of an input over a positive length of time is reflected in the emergence of interest, for interest is the reward for waiting.[7] Requirement (*i*) proves essential when there are heterogeneous goods. Requirement (*ii*) should be in effect even when there is only one homogenous good. In the case of a homogenous good, the problem of value is expressed in the form of the existence of interest. As for Process *b*, there is only one input—the foregone final good—and the standard of value is the final good itself; and the sole input is locked up in the process of production in no time. This implies that there is no room for interest to accrue. No problem of value exists here: the unit value of the capital good, measured in terms of the final good, is straightforwardly equal to ζ_i, the physical amount of the final good that has been foregone for that capital good. The value of k_i units of the *i*th capital good, measured in terms of the final good, is accordingly

(2) $$z_i = \zeta_i k_i$$

Expressed in value terms, N kinds of capital goods are summed to yield the aggregate measure of capital:

(3) $$K \equiv \sum_{i=1}^{N} z_i = \sum_{i=1}^{N} \zeta_i k_i$$

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variety of intermediate goods is the source of continuous growth, the problem of identifying different capital goods becomes crucial—and proves fatal (see Park, forthcoming).

[7] This concept of interest is neoclassical, of course. This should be the case as the present paper is tracking along the logic of neoclassical economics. The existence of interest necessary with a positive elapse of time is conceptual; interest, which must exist conceptually, may be zero in actuality. Also, in the setting of continuous time, one speaks of the ‘instantaneous’ rate of interest over an infinitesimally small interval of time; the nomenclature notwithstanding, the length of this ‘instantaneous’ or ‘infinitesimally small’ time is still larger than zero; hence, the existence of interest.

The aggregate 'quantity of capital' K, measured in terms of the final good, *is* the total amount of foregone final good.

By contrast, waiting exists—thus, time elapses—in the production of the final good using labour and capital goods (Process *a*). This process is represented by the production function:

(4) $Y = F(L, \mathbf{k}(N, k_i))$, or with the help of (3),

(4') $Y = F(L, K)$

The singleness of layer in this 'single-layered' economy refers to the feature of the economy that Process *a* is the sole process in which the dimension of time is positive.[8] In this process, thus, interest *must* accrue on capital, and this aspect is reflected in the following relationship:

(5) $Z = wL + \delta K + rK$

where w = the wage rate; L = aggregate employment of labour; δ = the (uniform) depreciation rate; r = the rate of interest; the sum Z of wages, depreciation and interest on capital constitutes the aggregate *value* of the final output, measured in terms of the final output itself.

Interest on capital in (5) can be construed in two different, but equivalent, ways. If payment for the use of capital is made at the beginning of the production period (*ante factum* payment), interest on capital represent the opportunity cost for the user of the capital: the fund she had spent on capital could instead have been used for lending, thereby obtaining interest. Here, interest is the reward for the waiting of the user of the capital goods. If payment is made at the end of the production period (*post factum* payment), interest now represents the reward for the waiting of the provider of capital goods.[9] The capital goods are provided at the beginning of the period, thus incurring current cost to their provider at the beginning of the period (this current cost is K); however, as he is waiting until the end of the period to get paid for renting out these capital goods, his waiting should be compensated for by interest on the current cost.

[8] In Figure 1 (and also in Figure 2), the positive elapsing of time is pictured by a solid arrow and the opposite case by a dotted line.
[9] This case fits better into the understanding of interest as the reward for the foregoing of consumption.

Either way, the result is the same: the existence of interest in (5).

The final good, understood as the physical quantity, is either used for consumption or foregone for investment, the foregone output being in turn used for replenishing used-up capital goods (F_1) and increasing the stock of capital goods (F_2):

(6) $$Y = C + F_1 + F_2$$

The construal of the 'transformation' of capital goods from the final good leads to

(7) $$F_1 \equiv \sum_{i=1}^{N} \delta\zeta_i k_i = \delta K \quad \text{and} \quad F_2 \equiv \sum_{i=1}^{N} \zeta_i \dot{k}_i = \dot{K}$$

(henceforth, a dot over a variable denotes the time derivative of the variable). The physical output of the final good which is used in replenishing used-up capital goods, understood in terms of physical quantities, is identically equal to the depreciation in the 'quantity of capital', understood in terms of value. The same is true of the final good foregone for net investment and the increase in the 'quantity of capital'. One is also led to

(8) $$K(\tau) \equiv \int_0^{\tau} \dot{K}(t)dt = \int_0^{\tau} F_2(t)dt$$

That is, the value of the capital stock is the same as the accumulated foregone final good. One notes that the vexing problem of measuring 'capital' and 'output' does not arise here.

Thanks to this, the use of the final good, understood as physical quantity, is represented by the following relationship:

(9) $$Y = C + \delta K + \dot{K}$$

Z in (5) is the value of the final output measured in terms of itself; thus it is equivalent to a certain number of the final goods. Y in (9) is the physical quantity of the final output. The national accounting requires that the two must be equal. One thus has

(10) $$wL + rK = C + \dot{K}$$

3. The vanishing of time: horizontal innovation models

The scene changes with horizontal innovation models—or, does it? We shall take the model of Barro and Sala-i-Martin (2004, Ch. 6) as our reference model of the horizontal innovation literature.[10] The production process of this economy is schematized as Figure 2.

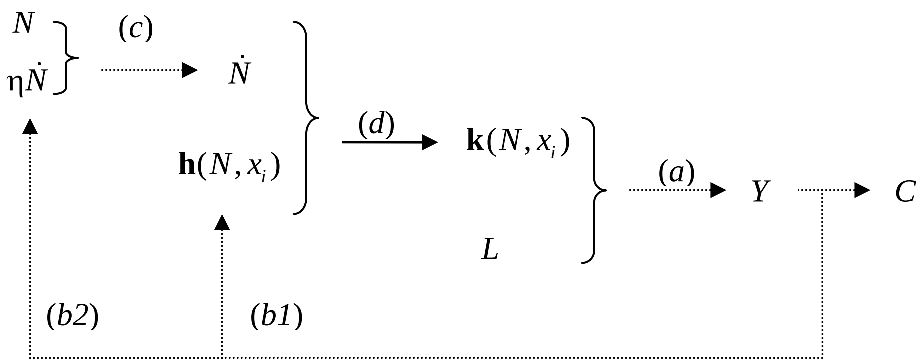

Figure 2: The production processes in an economy *à la* Barro and Sala-i-Martin (2004)

η = the quantity of final output required to produce a design;
$\dot{N}$ = the number of new designs;
N = the total number of designs;
x_i = the (physical) quantity of the ith intermediate good;
$\mathbf{g}(N, x_i)$ = the N-vector of quantities of final output required to produce N intermediate goods;
$\mathbf{k}(N, x_i) = (x_1, x_2, ..., x_n)$ = the N-vector of quantities of N intermediate goods.

In this economy, Process a is represented by the production function of the final good sector:

[10] The reader can leisurely check that the same analysis applies to other representative models of horizontal innovation, such as Romer (1990) where the 'accounting measure of capital' is defined in such a way that it is both the physical quantity of 'foregone' final good and the 'quantity of capital' on which interest accrues (which thus must be a value term).

(11) $$Y = f(L, \mathbf{k}(N, x_i))$$

which usually takes the 'Dixit-Stiglitz' form:[11]

(12) $$Y = L^{1-\alpha}\sum_{i=1}^{N} x_i^{\alpha}$$

In accordance with this production function, one has gross national income as the sum of the payments for the use of the factors of production, which thus must be in value terms:

(13) $$Z = wL + \sum_{i=1}^{N} \rho_i x_i$$

where w = the wage rate; ρ_i = the gross rental rate on the *i*th capital good. The *numéraire* of the economy is the final good. As x_i is a physical quantity of a good which is different from this *numéraire*, the rental rate of the *i*th intermediate good involves the price of that good relative to the final good. Thus,

(14) $$\rho_i = (\delta_i + r_i)\, p_i$$

where δ_i is the depreciation rate, r_i the rate of interest and p_i the price of the good in terms of the *numéraire*.

As it turns out, it is assumed that $r_i = 0, \quad \forall i$. Barro and Sala-i-Martin seem to think that this follows from their explicit assumption that perfect competition prevails in the final good market. But what this latter assumption implies is no more that that long-run equilibrium will prevail so that the rates of profit on the respective capital goods are uniform. For the case where $r_i = 0, \quad \forall i$, one needs an additional assumption: the production of the final good is *immediate*—there is *no* time elapsing from the purchase and application of the inputs to the production and selling of the output.

If instead there is a time gap between input and output (so that the unit period of

[11] One characteristic of this function is that intermediate goods x_i are 'symmetric' to each other; that is, they enter into production in a non-differentiated way—a change by one unit of any intermediate good brings about an identical change in production, as far as the existing stocks of the intermediate goods are the same. This symmetry leads to an identical demand function for all the intermediate goods.

production is positive), interest *must* occur. Similarly with the 'single-layered' economy above, this is the case regardless of whether the producers of the final good (who are the purchasers of intermediate goods) pay for the use of the intermediate goods at the beginning or at the end of the production period. If the payment is made *ante factum*, the producers of the final good must in equilibrium recoup the opportunity cost incurred on that payment over the production period—this is the interest on the payment for the intermediate goods. If the payment is made *post factum*, such opportunity cost is none for the producers of the final good. However, then, the producers of intermediate goods are instead subject to an opportunity cost. This opportunity cost must be represented by interest on the revenue which would have accrued if payment were made immediately with the provision of the goods. Long-run equilibrium in the sector of intermediate goods then requires that the producers of the intermediate goods should charge the purchasers the price which included this opportunity cost. Thus, whether payment is made *ante factum* or *post factum,* the result is the same: the rental price of intermediate goods should include interest *unless* one makes the assumption that no time is required for production.

Barro and Sala-i-Martin makes an additional, non-essential, assumption that intermediate goods are non-durable so that they are used up in a unit period of production: $\delta_i = 1$. Thus one has

(14') $\rho_i = p_i$

It is assumed that the production of one unit of the *i*th intermediate good requires, on top of the *i*th design, a uniform amount θ of the final output;[12] thus,

(15) $\mathbf{h}(N, x_i) = (\theta x_1, \theta x_2, ..., \theta x_n) = \theta \mathbf{k}(N, x_i)$

This uniformity of the production technique for the respective intermediate goods yields an identical supply function. One consequently has a 'symmetric equilibrium' where

(16) $x_i = x, \quad \forall i$

The same situation also implies that

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[12] As for the unit of an intermediate good, a more precise statement is that the unit of each intermediate good is so *defined* as to require θ units of final good for the production of one unit of that intermediate good.

(17) $p_i = p, \forall i$

Then (13) is reduced to

(18) $Z = wL + pxN$

The value of an intermediate good, $p_i x_i$, measured in terms of the final good, is obtained through the 'arbitrage equation' (Jones, 1998) for the intermediate goods sector. With the assumptions that labour supply is constant and that intermediate goods are nondurable, one has

(19) $rP_R = p_i x_i - \theta x_i$, or, using the results (16) and (17),

(20) $rP_R = px - \theta x, \quad \forall i$

where P_R is the price of a design. The right-hand side of (19) is the profit margin in an intermediate good firm in the same unit period; the left-hand side is the interest cost on a design accruing in a unit period. An arbitrage in the use of fund between purchasing a design and using it in production (thereby obtaining profit) on the one hand and purchasing a design and renting it (thereby obtaining interest) on the other requires that in equilibrium the return in either use of the fund be equal.[13]

Here one should not fail to notice an important aspect: time runs, for however short an interval (an 'instant'), in the production of intermediate goods. The existence of

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[13] Rearranging (20) gives

(20') $P_R = \dfrac{px - \theta x}{r}$

The left-hand side is the purchasing price of a design. The right-hand side is the present value of the flows of 'profits' over perpetuity. The two must be equal for equilibrium in an intermediate good sector. Still another rearrangement of (20) enables us to construe it from the perspective of costs.

(20'') $px = rP_R + \theta x$

The right-hand side stands for the total costs of producing a type of intermediate goods by the amount of x. A design is a durable good, so that the cost of purchasing it (P_R) is spread over perpetuity; thus, in each 'round' of production of an intermediate good, the cost of using the design is a fraction r of P_R.

interest on a design is the proof. If the production of an intermediate good were immediate, there would be no room for an arbitrage between profit and interest. For charging interest in compensation for renting a design presupposes a positive length, however short, of the renting period.

But this positive existence of time applies only to the design. In (20), the cost of the material input is measured of θ. θ is at first given as an engineering constant, the physical amount of the final good required for the unit production of an intermediate good. But the quantity to appear in (20) must be the *value* of this input. Suppose that the intermediate-good-producing firm pays for the final good input at the beginning of the production 'round'. If there is a positive elapse of time in production, the firm must be subject to additional cost, that is, the cost of waiting: interest. Thus the value of the final good input should be $(1+r)\theta$ (the depreciation rate of the final good is 1). Suppose by contrast that the payment for the use of the final good input is made at the end of the period. Then the cost of waiting is incurred to the provider of the input, and she—aware of this fact—will charge the purchaser the price which takes account of this cost of waiting. The price should be $(1+r)\theta$. The formulation (20) is *internally inconsistent* in the matter of treating time. This poses, as always with any internal inconsistency, a serious problem for horizontal innovation models, as internal inconsistency deprives them of one most effective defence. Some may argue that horizontal innovation models in fact take account of production time in all the sectors so that, at least conceptually, interest exists; it is only that they assume zero rate of interest on the final good input and the intermediate goods. But this argument goes against the long-run equilibrium condition of a uniform rate of interest, for *there is a positive rate of interest on a desig*n.

Profit maximization in the R&D sector yields the following relationship:

(21) $$P_R = \eta$$

where η is the amount of the final good required to produce a design. By now it will be too easy to see that an assumption is working here that a design is produced immediately with the application of the final good as input. (Even though the stock of previous designs is also used as an input, it incurs no marginal cost for it is a public good.)

One thus has, from (20) and (21),

(22) $$px = r\eta + \theta x$$

Substituting (22) into (18), one has

(23) $$Z = wL + r\eta N + \theta xN$$

This is a counterpart of the 'single-layered' case (5): the aggregate income is composed of the aggregate wage, the aggregate interest on the 'assets' of the economy (which is the stock of designs, valued in terms of the final good) and the full depreciation of the intermediate goods.

Meanwhile, Processes *b1* and *b2*, taking account of (16) and (17), lead to

(24) $$Y = C + F_1 + F_2$$, with

(25) $$F_1 = \theta xN \quad \text{and} \quad F_2 = \eta \dot{N}$$

This is analogous to the 'single-layered' case, represented by (6) and (7): the final output is used for consumption, the replenishment of the used-up intermediate goods and the increase in the 'assets' of the economy, all the magnitudes understood as physical quantities of the final good.

In both (23) and (24), all the magnitudes are measured in terms of the final good. In (24), $F_1 (= \theta xN)$ is the amount of the final good which is foregone for the production of intermediate goods; in (23), θxN stands for the value of the used-up intermediate goods; they must be equal. Similarly, in (24), $F_2 (= \eta \dot{N})$ represents the amount of the final good which is foregone for the increase in the number of designs; in (23), ηN stands for the value of the stock of (durable) designs, on which interest is ensued; thus, the value of the stock of the durable assets of the economy (*A*), measured in terms of the final good, is straightforwardly the accumulated final good which is foregone for its production:

(26) $$A(\tau) \equiv \eta N(\tau) = \int_0^{\tau} \eta \dot{N}(t) dt$$

The value *Z* of the final output, measured in terms of itself, and the physical quantity *Y* of the final output must be equal. Thus, one has

(27) $$wL + r\eta N = C + \eta \dot{N} \quad \text{or} \quad wL + rA = C + \dot{A}$$

One immediately notes that this is an exact counterpart of (10) in the case of the 'single-layered' economy. The only difference is that now the stock of 'assets' of the economy is the stock of designs ($A \equiv \eta N$) whilst previously it was the stock of capital goods (*K*).

It is by a sleight of hand that the Barro and Sala-i-Martin model (or horizontal innovation models in general) has essentially the same feature as the traditional neoclassical model regarding the time structure of the economy. In the traditional neoclassical model, there is (explicitly) only one layer of production, which involves a lapse of positive time: from the application of the final good as the investment good until the appearance of the final good as output; by contrast, there is no positive time involved in the transformation of the final good into the investment good. In horizontal innovation models, there are potentially three 'sectors' of production. However, the number of production layers is reduced to one: no time runs either in the final good sector or in the R&D sector. A positive length of time passes only in the intermediate goods sector, from the instant of applying designs as one of the inputs till the point of intermediate goods being produced as the output (bizarrely, even in this sector, time does not apply to the other of the two inputs, the final good input). The potentially 'three-layered' economy is reduced to a 'single-layered' one—not on any economic ground but solely by pure assumption, in the name of modelling (though unwittingly). In the world of horizontal innovation, time is vanished partially – hence, inconsistently.

4. The return of time: a 'multi-layered' economy

Time discriminates neither in reality nor in logic. Production takes time—this is reality. If production in one sector of the economy takes time, then production in the other sectors must take time, too (especially if this 'production' is the process of 'real' transformation, that is, transforming inputs into an output which is heterogeneous from the inputs); moreover, if one of inputs in a sector takes time to be used, then other inputs in the same sector must take time to be used, too—this is logic. The treatment of time in horizontal innovation models is at odds with reality and violates logic. Reality and logic dictate a positive and undiscriminating existence of time in production.

In the case of the 'single-layered' economy, the 'transformation' of the final good into capital goods in a timeless setting *does* make sense. Here, the final good is in itself usable for multiple purposes; thus, if it is not used for consumption—that is, if it is foregone—then it is automatically used for investment. Foregone output is *in itself* an investment good; the input and the output are identical. However, the situation is different in horizontal innovation models. In the R&D sector, the input is the final good and the output is a design; they *are* different things. This means that some process of *real* transformation must exist; with reality, time must come in. Process *c* in Figure 2, to be real, must take place in time. Similarly in the final good sector, the inputs are labour and intermediate goods and the output is the final good. Heterogeneity between the inputs and the output should require, again, some process of real transformation—and thus time. Process *a* in Figure 2, to be real, must take place in time. Recall that this process's counterpart in the 'single-layered' economy (Process *a* in Figure 1) does take place in time.

Note further that the only sector in horizontal innovation models in which production takes time (Process *d* in Figure 2) is the counterpart of the production process in the 'single-layered' economy where transformation is done immediately (Process *b* in Figure 1)—and we have said just above that the treatment of time for this sector in the 'single-layered' economy (that is, *no* time) makes sense. This sense-making must have had some appeal to authors of horizontal innovation. Observe how they comment regarding the production of intermediate goods—the only production process which takes time in their model—that '[i]n effect, the inventor of good *j* sticks a distinctive label on the homogenous flow of final output and, thereby, converts this product into the *j*th type of intermediate good' (Barro and Sala-i-Martin, 2004, p. 291); or in more honest if cruder words, '[o]nce the design for a particular capital good has been purchased (a fixed cost), the intermediate-goods firm produces the capital good with a very simple production function: one unit of raw capital can be *automatically* translated into one unit of the capital good' (Jones, 1998, p. 104, emphasis added).[14] By describing production as involving only 'sticking labels' or as 'automatic translation', they almost or strongly suggest that the production of intermediate goods is timeless.[15] But, on the other hand, production in this sector involves the other input—designs; hence, a process of real transformation. The (unwitting) solution is to treat time

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[14] All these authors set $\theta = 1$, perhaps in consistence with their feeling of what actually happens in Process *d*. Recall that in the 'single-layered' economy, the usual—and conceptually the more consistent—setting for Process *b* is $\zeta_i = 1, \quad \forall i$; see footnote 6 above.

[15] This arouses a suspicion that label-sticking or automatic translation indeed takes place in a timeless setting, just as in Process *b* in the 'single-layered' economy, implying that intermediate goods are in fact undifferentiated from each other and also from the final good input. Park (forthcoming) argues that, in the current modelling of horizontal innovation, there is nothing in the models that identifies different intermediate goods.

schizophrenically: a design takes time to be used whilst the final good input takes no time.

Time should be restored in full to all the sectors and to all the (material) inputs. Equations representing the economy must be correspondingly modified. Equations representing production, with full account of time, can be constructed either on the assumption that payment for the use of all inputs is made at the end of the production period (*post factum*) or on the assumption that payment for the use of inputs, except for labour,[16] is made at the end of the period (*ante factum*). The result does not hinge a jot upon which assumptions. The following takes the first assumption; though this will make the construction more cumbersome than taking the second assumption, it has the advantage of putting the underlying matter in much shaper relief.

Production of the final good uses labour and intermediate goods. Payment for them is made *post factum*. Thus, the ‘price equation’ for the final good sector remains untouched from Barro and Sala-i-Martin’s own formulation.

$$(18^*) \qquad Z = wL + pxN$$

(Recall that intermediate goods are non-durable, so that their depreciation rate is unity.)

Time that passes in the production of the final good is reflected in the determination of the price of an intermediate good. The producer of an intermediate good provides the good at the beginning of the production period, knowing that she will get paid at the end of the period. For her, this waiting involves opportunity cost and must be compensated for; if not, this is equivalent to assuming zero time for the production of the final good. The price which the producer of the intermediate good charges to its purchaser must reflect this opportunity cost. The current cost of production of an intermediate good consists of interest cost on a design and the payment for the final good input; this current cost should attract some interest with the passage of time. Thus, the ‘price equation’ for a type of an intermediate good produced by the amount of x is

$$(20^*) \qquad px = (1+r)(rP_R + \xi x), \quad \forall i$$

(An intermediate good is non-durable, so that the depreciation rate is unity.)

[16] The different treatment of material inputs (capital goods) and labour (and land) is of course in accordance with the economic reasoning which has resulted in the classification of the ‘factors of production’ into the three categories.

Note that the current cost of the final good input is expressed by ξ, not by θ. θ is the physical amount of the final good which is used in the production of one unit of an intermediate good; ξ is its *value* (price). When the final good is transformed into an intermediate good, θ units of the former are used for one unit of the latter. If the resulting intermediate good gets paid immediately with the start of production process, its value which must be redeemed to the provider of the input is equal to θ per unit. But when the intermediate good gets paid after a unit period of production, its value to be redeemed must take account of the passage of time; that is, interest. And, of course, the *cost* of production should be measured in terms of value, not merely in terms of physical quantities. Thus, one has (recalling that the final good as the input is non-durable)

(28) $\xi = (1+r)\theta$

The situation is similar in the R&D sector. Here one new unit of design requires a physical quantity η of the final good. If this input gets paid at the end of the production period, its value must include the cost of waiting. As the stock of previous design is free currently and the final good input is non-durable,

(21*) $P_R = \upsilon$

(29) $\upsilon = (1+r)\eta$

where υ stands for the value of the final good input in the R&D sector.

We now have the counterpart of (18) above:[17]

(23*) $Z = wL + r\left[(1+r)^2 \eta N\right] + \left[(1+r)^2 \theta x N\right]$

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[17] The same expression is obtained even if one assumes the *ante factum* payment for inputs (except for labour). The counterpart 'price equations' will be

(18**) $Y = wL + (1+r)\,pxN$

(20**) $px = rP_R + (1+r)\theta x, \quad \forall i$

(21**) $P_R = (1+r)\eta$

In this case, r represents the opportunity cost of waiting of the *users* of the inputs in the respective sectors. In the case of *post factum* payment, by contrast, p in (18*), ξ in (20*) and υ in (21*) must reflect the opportunity cost of waiting of the *suppliers* of the corresponding inputs, which is thus represented in (20*), (28) and (29), respectively.

This is the result obtained by going through the three 'layers' of production, all 'layers' being associated with a positive length of time. The magnitude in the first square brackets on the right-hand side of (18*) is the *value* of the stock of designs measured in terms of the final good; that in the second square brackets is the aggregate *value* of the intermediate goods which are used up in production.

The national accounting for the use of the final good must be the same as before, for it refers to the relationship among physical quantities, without involving time:

(24) $Y = C + F_1 + F_2$, with

(25) $F_1 = \theta xN$ and $F_2 = \eta\dot{N}$

With time reinstituted in production, there holds no longer the quantitative identity either between the forgone output for the production of designs and the value of their stock, or between the foregone output for intermediate goods and the values of their stock. Time drives a wedge between the physical quantity of an input that is actually expended and its value. The total amount of the foregone final good for intermediate goods is $F_1(=\theta xN)$, whilst its aggregate value in the production of the final good, measured in terms of the final good, is $(1+r)^2\theta xN$. The interest factor in the value term reflects the length of time which has elapsed, first, from the foregoing of consumption to the production of intermediate goods and, second, from their use in the production of the final good and the payment for their use. The same reasoning applies to the case of designs. The amount $F_2(=\eta\dot{N})$ of the final good is foregone as an input for the production of designs. As this amount accumulates, the stock of designs grows to:

(30) $\eta N(\tau) = \int_0^\tau F_2(t)dt$

But as the input ηN of the final good goes through the three stages of production, interest accrues on their stock correspondingly three 'times'—and then the value of the stock of designs (A) diverges from the accumulated final good foregone for the designs.

(31) $A(\tau) \equiv \left[(1+r)^2\eta N\right] \neq \int_0^\tau \eta\dot{N}(t)dt$ or $\dot{A}(\tau) \neq F_2$

The reader who is versed in capital theory will not fail to note that this will cause all sorts of problems that are well-known in that area of economic theory.

However, this divergence between value and quantity cannot be the case for the value Z of the total final output and the total physical output Y of the final good. Recall that Z is measured in terms of the final good itself and thus expressed as a certain quantity of the final good; this quantity must be equal to the physical amount that is produced, that is Y. Thus, one should have

$$(27^*) \qquad wL + r\left[(1+r)^2 \eta N\right] + \left[(1+r)^2 \theta x N\right] = C + \eta \dot{N} + \theta x N$$

which some manipulation will transform into

$$(27^{**}) \qquad wL + \left[r(1+r)^2 + r(2+r)\eta^{-1}\theta x\right]\eta N = C + \eta \dot{N}$$

Contrast with (27) is conspicuous.

If the representative household maximises discounted utility over infinite lifetime on the basis of the utility function

$$(32) \qquad U(t) = \frac{C(t)^{1-\sigma} - 1}{1-\sigma}$$

with (27**) as the constraint, the resulting Euler equation is

$$(33^*) \qquad \frac{\dot{C}}{C} = \frac{\phi - \rho}{\sigma}$$

where $\phi \equiv r(1+r)^2 + r(2+r)\eta^{-1}\theta x$ and ρ is the rate of time preference. Of course, for Barro and Sala-i-Martin, who use (27) as the constraint, the Euler equation is

$$(33) \qquad \frac{\dot{C}}{C} = \frac{r - \rho}{\sigma}.$$

5. The revenge of time: the 'collapse' of equilibrium

The present paper might be much ado for nothing if the full-time economy produced qualitatively the same conclusions as the part-time economy. The latter would be acceptable or even better because it enabled simpler calculation of equilibrium values. From the former's perspective, the latter might seem entitled to claim to be a better reflection of reality, but adding nothing substantial to the understanding of essential features of endogenous growth. But this is not the case.

From maximising the profit of the final good sector, one gets the equality between the price of an intermediate good and its marginal product:

$$(34)\quad p_i = \alpha L^{1-\alpha} x_i^{\alpha-1}$$

Profit maximisation in the intermediate goods sector yields the following 'monopoly pricing' rule:

$$(35)\quad p_i = \alpha^{-1}\theta(1+r)$$

As we have discussed above, the 'price equation' for the intermediate goods sector is

$$(36)\quad p_i x_i = (1+r)\left[r(1+r)\eta + (1+r)\theta x_i\right]$$

These three relationships constitute an independent system with the corresponding three unknowns: *p*, *x* and *r* (by virtue of (35), one can drop the subscript *i* from p_i and x_i: the equilibrium is symmetric.). One ends up with the following equation in *x*:

$$(37)\qquad \left(\alpha^{-1}\theta x\right) - \left(\alpha^{2} L^{1-\alpha} x^{\alpha}\right) + \left(\eta\alpha^{2}\theta^{-1} L^{1-\alpha} x^{\alpha-1}\right) - \left(\eta\alpha^{4}\theta^{-2} L^{2(1-\alpha)} x^{2(\alpha-1)}\right) = 0$$

In general, this equation cannot be solved algebraically. Moreover, the right-hand side of the equation can be a non-monotonic function of *x*. This implies that the solution may not exist; that even if the solution exists, it may be multiple; that even if the solution exists uniquely, it may not be meaningful, being negative.[18]

This threatens one most important element that makes sense of an equilibrium approach such as the horizontal innovation literature: the guaranteed existence, uniqueness and meaningfulness of equilibrium. Without the guarantee of these characteristics of equilibrium, the model loses its robustness, theoretical or empirical.

6. Epilogue: *Tempus regnat et non discriminat*

A model abstracts. Abstraction involves a removal of what is considered as inessential for the insight a model wishes to convey. Thus the degree of 'realism' of a model may not be a strong criterion of the 'validity' of a model; it may be a matter of methodology. But there is an absolute criterion for the validity of a model: *internal consistency*.

The horizontal innovation model discussed above commits the error of internal inconsistency by applying time discriminatively. The discrimination is committed doubly: first, time elapses positively in production in the intermediate goods sector whilst production takes place in no time in the remaining two sectors of the economy; second, in the intermediate goods sector, only one of the two inputs—a design—is subject to time whilst the other input—the final good input—is free from time. When time is duly reinstituted in all sectors and for all inputs, the result is the threatening of one most important element that makes sense of an equilibrium approach: the existence, uniqueness and meaningfulness of equilibrium.

Internal inconsistency is not confined to the model in question. Different models use different model settings (assumptions), but they invariably incur internal consistency of the kind similar to the one disclosed above.[19] This is because the economy that horizontal innovation models wishes to deal with is a 'multi-layered' economy whilst they must reduce it effectively to a 'single-layered' economy in order to avoid the problem of value necessarily arising in a 'multi-layered' economy; in this

[18] Barro and Sala-i-Martin (2004) has, instead of (35) and (36),

(35') $p_i = \alpha^{-1}\theta$

(36') $p_i x_i = r\eta + \theta x_i$

Together with (34), the system solves for *p*, *x* and *r* uniquely and sensibly.

[19] In Romer (1990), internal inconsistency is crudely posed: time is duly taken into account in the design sector and the intermediate goods sector but this account is completely brushed away by the 'accounting measure of capital' in the final good sector. The working out of the critique of Romer's case is available on request from the present author.

process of reduction, time is forced to apply discriminatively.

A spectre of value and capital haunted economics. It still does. And it will, as far as production takes place in time. *Tempus regnat et non discriminat*.

References

Aghion, Philippe and Hewitt, Peter (1992), ‘A model of growth through creative destruction’, *Econometrica*, vol. 60, 323–351

Barro, Robert and Xavier Sala-i-Martin (2004), *Economic Growth*, 2nd edition, Cambridge, MA and London: The MIT Press

Bénassy, Jean-Paul (1998), ‘Is there always too little research in endogenous growth with expanding product variety?’, *European Economic Review*, vol. 42, 61–69

Gancia, Gino and Fabrizio Zilibotti (2005), ‘Horizontal innovation in the theory of growth and development,’ *Handbook of Economic Growth*, eds. Philippe Aghion and Steve Durlauf, North Holland, Amsterdam

Jones, Charles (1998), *Introduction to Economic Growth*, New York and London: W. W. Norton & Company

Park, M.-S. (forthcoming), ‘Homogeneity masquerading as variety: the case of horizontal innovations models’, *Cambridge Journal of Economics*

Romer, Paul M. (1987), ‘Growth based on increasing returns due to specialization’, *American Economic Review*, vol. 77, 56–62

Romer, Paul M. (1990), ‘Endogenous technological change’, *Journal of Political Economy*, vol. 98, S71 – S102

Solow, Robert, M. (1956), ‘A contribution to the theory of economic growth’, *Quarterly Journal of Economics*, vol. 70, 65–94