

## WAGES, PROFITS AND “EXPLOITATION”: A NEORICARDIAN FORMULATION

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This paper suggests a development of the Ricardian approach to value and distribution, with respect to the relation between wages and profits and its association to the relation between “necessary labour” and “surplus labour”, or what Marx called “exploitation”<sup>2</sup>. The argument recalls Ricardo’s formulations, formalizing them in the manner of Sraffa (1960), and then follows Sraffa in relaxing the Classical treatment of the wage as a “basket” of goods, required for the subsistence of the worker. Proceeding in the Sraffian scheme, it connects the wage-profit relation to “exploitation” as a relation between “paid labour” and “unpaid labour” (recalling another definition by Marx). However, “paid labour” is redefined by means of a concept of purchasing power of the aggregate amount of wages over a share of the Net National Product; while “unpaid labour” is redefined accordingly. There emerges a simple and meaningful concept of “exploitation”, which does not depend on Marxian “values” or “surplus value”, nor does it depend on a reduction of heterogeneous (or “complex”) labour to homogeneous (or “simple”) labour. The “rate of exploitation” comes out as a “shadow” ratio determined as a *result* of the distribution of national income between labour and capital, or rather as a simple function of the labour share.

Ricardo’s theory about necessary labour and surplus labour appeared in his *Principles of Political Economy* (1817) particularly in a reference to changes in the general rates of wages *and profits* as being accompanied by variations in the “proportion of the annual labour of the country [that] is devoted to the support of the labourers” (Ricardo, 1817, p.49). Marx (1867) followed Ricardo in this formulation<sup>3</sup> but he developed his theory of wages and profits in terms of his own concept of “value” (assuming exchange values of commodities to be

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<sup>2</sup> “exploitation is defined as the appropriation by a certain class of the productive labour of another class, without the appropriators themselves supplying to the other class an equivalent counterpart in terms of productive labour. ... exploitation is not peculiar to capitalism. It runs through all class-divided societies of the past...” (Catephores, 1989, p.58)

<sup>3</sup> “Just as the individual labourer can do more surplus-labour in proportion as his necessary labour-time is less, so with regard to the working population. The smaller the part of it which is required for the production of the necessary means of subsistence, so much the greater is the part that can be set to do other work.” (Marx, 1867, p.513; emphasis added)

proportional to the quantities of labour necessary to produce them). Being so developed, his theory has got into the trouble of “transformation” of “values” to “prices of production”<sup>4</sup>; and his “values” have shown after all to be irrelevant in the quantitative determination of prices of production, wages and profits<sup>5</sup>. However, as will be seen below, Marx himself had made a distinction between, on the one hand, his “rate of surplus-value” (a ratio between “surplus-value” created by using “labour-power” and the “value” of the latter) and, on the other hand, his “rate of exploitation”; the latter being defined as a ratio between “surplus labour” and “necessary labour”, or between “unpaid labour” and “paid labour”. These variables (quantities of labour) and the corresponding concepts of exploitation do not depend on Marx’s “values”, and can be incorporated into the Ricardian formulations, thus ridding them of the *imbroglio* with the Marxian theory of “value”.

The first of such concepts of exploitation – in terms of “surplus labour” and “necessary labour” – is the same as the one Ricardo had already formulated (though not using the term “exploitation”), as will be seen in section 2 below. The second concept – in terms of “unpaid labour” and “paid labour” – has been developed in Marx’s *Capital* into a more general concept (from a relaxation of the supposition of a subsistence wage basket), as will be shown in section 5, where it is modified and then incorporated into the reformulation made by Sraffa of Ricardo’s system.

Sections 1 and 2 present Ricardo’s formulations in the *Essay on Profits* (1815) and in the *Principles of Political Economy* (1817). Sraffa’s version of Ricardo’s theory is presented in section 3, and his own basic model in section 4. Section 5 submits a “Neoricardian” or “Sraffian” development of the theme. The last section suggests some possible generalizations or extensions, and concludes with a few comments.

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<sup>4</sup> “Prices of production” are the same as the “natural prices” of Adam Smith, which were also taken up by Ricardo and Sraffa, as will be seen below.

<sup>5</sup> See Steedman(1977); see also Samuelson(1974), where he argues that the recourse to “values” by Marx was an “uninsightful detour”.

## 1 The “corn model” in Ricardo’s *Essay on Profits* (1815)

The “corn model”, an “agricultural” theory of profits implicit in the *Essay on profits* (Ricardo, 1815) has been the first and most abstract of a class of models which expose the nature of profits as “surplus product” and “surplus labour”, while determining the magnitude of profits and establishing its inverse relationship with real wages<sup>6</sup>.

The model assumes there is an abundance of land, which is homogeneous and free, so that there is no “rent of land” (rental payment for its use)<sup>7</sup>. It also assumes that production is done with the employment of labour solely, i.e. with “unassisted labour” (without means of production such as materials and equipment). The production of all commodities (goods and services) takes an yearly cycle, the wages of the labourers being advanced by capitalists at the beginning of the year. At the end of the year, every commodity is sold at a “market price” which is supposed to be equal to its “natural price”<sup>8</sup>. This price is just sufficient to replace the amount of wages which had been advanced (and which makes the “circulating capital” that was invested over an yearly cycle) together with an amount of profits on such capital at a uniform rate across all branches of production, one for each commodity.

This model may be formalized as a simple set of equations for the determination of the “natural” prices, together with an equation which determines the “natural” wage rate<sup>9</sup>:

$$\begin{aligned} p_a &= l_a w(1+r) \\ p_b &= l_b w(1+r) \\ &\dots\dots\dots \\ p_k &= l_k w(1+r) \end{aligned} \tag{I}$$

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<sup>6</sup> The intellectual formation of this theory, on the part of Ricardo, was reconstructed in Macedo(1990), with some references to personal and historical circumstances. An extract of that work is presented in Macedo(1991).

<sup>7</sup> “land equally fertile, and equally well situated, might be abundant [and might “be had by any one who chooses to take it”]” (Ricardo,1815,p.11[and 10]); “no rent could be paid for such land [“when there was an abundant quantity not yet appropriated”], for the reason stated why nothing is given for the use of air and water, or for any other of the gifts of nature which exist in boundless quantity” (Ricardo, 1817,p.69).

<sup>8</sup> See Smith (1776),Bk.I,ch.7 “Of the natural and market Price of Commodities”

<sup>9</sup> Ibid., ch.8 “Of the Wages of Labour”

$$w = a \quad (II)$$

where the subscripts **a,b,...,k** indicate the commodities, **a** being “corn” or “food”, and **b** being “cloth” or “clothes”; **p<sub>a</sub>,...,p<sub>k</sub>** are the prices or values of commodities, expressed in terms of corn, the price of which is identically 1, i.e. **p<sub>a</sub>≡1**; **l<sub>a</sub>,...,l<sub>k</sub>** are *given* quantities of labour required directly for the production of a unit of the commodities **a,b,...,k** respectively, taking as the unit of labour a “man-year”, i.e. the quantity of labour performed by a worker in a given lapse of time (measured in hours, for example) during the year, with a given intensity of work (quantity of labour per unit of time); **r** is the general (uniform) rate of profit; **w** is the rate of wages, i.e. the wage per unit of labour, being expressed in terms of corn; and **a** is a given quantity of corn that is necessary for subsistence per man-year.

The rate of profit in agriculture (i.e. in the production of “corn”), **r<sub>a</sub>**, is *defined* thus:

$$r_a \equiv (p_a - l_a w) / l_a w$$

being equal to **(1 - l<sub>a</sub>a) / l<sub>a</sub>a**, according to equation II. The latter expression can be read as the inverse ratio between the quantity of corn consumed as means of subsistence, or “necessary consumption” (per unit of product), and the remaining quantity of corn (per unit of corn produced), or “surplus product”.

Since **l<sub>a</sub>** and **a** are given, and **w=a**, the rate **r<sub>a</sub>** is then determined, independently of **r** and **p<sub>i</sub>** for **i=b,...,k**. But the first of equations I imply **r=r<sub>a</sub>**, and therefore **r** is determined by **r<sub>a</sub>** or, in Ricardo’s words, **r** is “regulated” by **r<sub>a</sub>**. This **r<sub>a</sub>** is the rate of profit in the production of corn or food, which is the sole “wage-good”. The remaining equations determine **p<sub>b</sub>,...,p<sub>k</sub>** such that **r<sub>b</sub>=...=r<sub>k</sub>=r=r<sub>a</sub>**.

Since **r=r<sub>a</sub>**, being **r<sub>a</sub>** dependent on **l<sub>a</sub>** and **a**, any variations of these parameters **a** and/or **l<sub>a</sub>** result in inverse variations of **r**. In particular, as Ricardo explains in the *Essay on Profits*:

“Supposing that the nature of man was so altered, that he required double the quantity of food that is now necessary for his subsistence, and consequently, that the expenses of cultivation were greatly increased... [leaving] a much less

*surplus produce*; consequently, the profits of stock could never be so high.”  
(Ricardo, 1815, p.15; emphasis added)

## **2 Ricardo’s model of value and distribution, in the *Principles of Political Economy* (1817)**

The principle of the “regulating” role of the profit rate that is determined in the production of the wage-good was generalized by Ricardo, in his *Principles* (1817), where it is extended to a wage basket containing “clothing” besides “food”; being thereby generalized in order to account for any number of wage-goods. His model of “food and clothing” can be described simply as an extension of the “food” model of his *Essay on Profits*, by extending the wage equation, which becomes then:

$$w = ap_a + bp_b \quad \text{(III)}$$

where **b** is a given quantity of “clothes” (or “cloth”). Ricardo’s model consists now of equations I and III<sup>10</sup>.

When Ricardo published the *Principles*, he had become aware that the rates of wages and profits, being supposed to be uniform, and to vary uniformly, in the production of all commodities, their variations would not affect the *relative* “natural” prices of commodities. But the costs of labour (wages advanced, or invested as capital) have different distributions over the successive stages of the production of the various commodities, or over the *time* required to bring them to market; so that the relative prices must change with variations in **r** (the uniform rate of return on capital). However, he thought, this is a “cause” of variations in relative prices of secondary importance, compared to changes in the quantity of labour required to produce each commodity, that is to say, changes in the productivity of labour in its production, due to technical innovations etc. So he decided to treat the relative prices of commodities as constants whenever the quantities of labour respectively necessary to produce

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<sup>10</sup> Equations I and III are a special case of a model formalized by Dmitriev (1898) from Ricardo’s argument in the *Principles*.

them remained constant – as if the commodities were produced with “unassisted labour” only (making abstraction of means of production), as in equations I above. In effect, taking any two of those price equations and canceling out the uniform rates  $w$  and  $r$ , it follows that  $p_i/p_j = l_i/l_j$ , for all  $i, j = a, b, \dots k$ .

Take now the modified wage equation III and the first two equations I, those concerning the production of the *wage-goods*. Together they form a sub-system which is sufficient to determine the relative price of the two commodities,  $p_b/p_a$ , and a profit rate common to both industries, which can be denoted by  $r_{a+b}$ , independently of  $r$  and of the remaining prices. Being  $p_b/p_a = l_b/l_a$ , the rate  $r_{a+b}$  comes to be:

$$r_{a+b} = [1 - (a_l a + b_l b)] / (a_l a + b_l b)$$

But equations I imply  $r = r_{a+b}$ , or that  $r$  is “regulated” by  $r_{a+b}$ , so that:

$$r = [1 - (a_l a + b_l b)] / (a_l a + b_l b) \quad (IV)$$

One can thus say with Ricardo that, given the wage basket  $\{a, b\}$ ,  $r$  is “regulated” by  $l_a$  and  $l_b$ , i.e. by the productivity of labour *in the production of wage-goods*<sup>11</sup>.

However, there is in the *Principles* a new and important reading of the “surplus product” in terms of “surplus labour”. In fact, Ricardo refers to changes in  $r$  as being accompanied by inverse variations in the “proportion of the annual labour of the country [that] is devoted to the support of the labourers” (Ricardo, 1817, p.49). As noted by Sraffa:

“It was now labour, instead of corn, that appeared on both sides of the account ... the ratio of the total labour of the country to the labour required to produce the necessaries for that labour.” (Sraffa, 1951, p.xxxii)

In effect, let  $N$  be the number of workers employed in the economy (i.e. in the whole set of industries) during the yearly cycle of production; and let  $L$  be the

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<sup>11</sup> “But suppose the price of silks, velvets, furniture, and any other commodities, not required by the labourer, to rise in consequence of more labour being expended on them, would not that affect profits? Certainly not: for nothing can affect profits but a rise in wages; silks and velvets are not consumed by the labourer, and therefore cannot raise wages.” (Ricardo, 1817, p.118)

quantity of labour (in men-years) performed by that whole set of workers, or what Ricardo calls (*loc.cit.*) “the annual labour of the country”. Multiplying the numerator and the denominator of expression IV by **N**, the denominator comes to be equal to the part of the “annual labour of the country” that is required to produce the means of subsistence **{Na, Nb}** which are necessary for all workers, or the aggregate “necessary labour”, now denoted by **L<sub>N</sub>**; and the numerator becomes the remaining part of the annual labour of the country, or “surplus labour”, denoted by **L<sub>S</sub>**. Expression IV then takes the aggregate form:

$$r = (L-L_N)/L_N \equiv L_S/L_N \quad (V)$$

Note in the expressions IV and/or V:

- (i) if **b=0** in expression IV, one has in retrospect the result of the “corn model”, **r=(1-a<sub>a</sub>)/a<sub>a</sub>**, with a new reading in terms of “necessary” and “surplus” quantities of *labour* (per man-year of labour, in that expression);
- (ii) in expressions IV or V, the right side is what Marx has called “degree of exploitation”<sup>12</sup>;
- (iii) the degree of exploitation is increased (or diminished) if **a** and/or **b**, and/or **I<sub>a</sub>** and/or **I<sub>b</sub>**, diminish (or increase);
- (iv) such changes in **a**, **b**, **I<sub>a</sub>**, **I<sub>b</sub>** imply, on the other hand, inverse variations of the general rate of profit.

Connecting (iii) and (iv), in Ricardo’s words: a higher “proportion“ of necessary labour in the total “annual labour of the country” corresponds to lower profits, as a consequence of “a rise of wages, from the circumstance of the labourer being more liberally rewarded, or from a [greater] difficulty of procuring the necessaries on which wages are expended” (Ricardo,1817,p.48)

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<sup>12</sup> Marx makes a distinction between the “rate of surplus-value”, which is a ratio between “values” (conceived as “materialized labour”), and the “degree of exploitation”, a ratio between quantities of “living labour”: “surplus-value bears the same ratio to variable capital, that surplus-labour does to necessary labour, or in other words, the rate of surplus-value  $s/v$ =surplus-labour/necessary labour. Both ratios,  $s/v$ , and surplus-labour/necessary labour, express the same thing in different ways; in the one case by reference to materialised, incorporated labour, in the other by reference to living, fluent labour” (Marx,1867,p.218). Marx sometimes refers to the “degree of exploitation” in aggregate form (for the whole set of workers), and this is then equivalent to Ricardo’s concept of a “proportion of the annual labour of the country [that] is devoted to the support of the labourers” (*loc.cit.*), which is also a ratio between quantities of “living” labour.

This immediate connection between the general rate of profit and Ricardo's "proportion"  $L_N/L$ , together with the inverse relationship between the rate of profit and the real wage (this being treated as a basket of wage-goods), depend on the simple theorem about value ( $p_i/p_j=l_i/l_j$ ) which belongs to this very abstract model, in which production requires "unassisted labour" only. But, in his *Principles*, Ricardo did not manage to demonstrate the inverse relationship between the wage rate and the profit rate, nor its connection to  $L_S/L_N$ , in a more general (less abstract) context of determination of relative "natural" prices (including means of production, fixed capital etc.); being troubled by the "second cause" of changes in such prices, namely variations of the rate of profit associated with variations of the wage rate.

### 3 Ricardo's model in Sraffa's *Production of Commodities* (1960)

After some work had been done by others, especially Dmitriev(1898), in order to formulate correctly Ricardo's theory, under more general suppositions than "unassisted labour", Sraffa (1960) succeeded in obtaining a rigorous formulation and generalization of the Ricardian relationships among wages, profits and natural prices. Part I of his book concerns the production of single commodities (there being no joint production) by means of labour and means of production, making abstraction of fixed capital and land. The determination of the general rate of profits, and its inverse relationship with the real wage rate, are there demonstrated enabling the relative prices of commodities (Smith's "natural" prices) to deviate from the relative quantities of labour required to produce them.<sup>13</sup>

The formulation by Sraffa of Ricardo's theory can be introduced here as an extension of system I+III by including means of production, besides labour, in the price equations:

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<sup>13</sup> Relative prices vary with the uniform rates of wages and profits because, in the production of the various commodities, the respective proportions between the value of the means of production and the quantity of labour employed are different and variable; and so also in the various stages of production of each commodity.



$$(a_a p_a + \dots + k_a p_k + l_a w)(1+r) = p_a$$

..... (VI)

$$(a_k p_a + \dots + k_k p_k + l_k w)(1+r) = p_k$$

$$w = a p_a + \dots + k p_k \tag{VII}$$

where  $a_a, \dots, k_k$  are (non-negative) requirements of each commodity per unit of each product (for *given* quantities of product in the respective industries,  $A, B, \dots, K$ , not yet shown in the equations) and  $\{a, \dots, k\}$  is a given subsistence basket, as in equation III (with  $a, \dots, k$  non-negative quantities, being some of them positive).

Multiplying both sides of equations VI by the gross quantities of products (supposed given) one obtains:

$$(A_a p_a + \dots + K_a p_k + L_a w)(1+r) = A p_a$$

..... (VIII)

$$(A_k p_a + \dots + K_k p_k + L_k w)(1+r) = K p_k$$

where  $A_a \equiv a_a A, \dots, K_k \equiv k_k K, L_a \equiv l_a A, \dots, L_k \equiv l_k K$ .

Substituting equation VII for  $w$  in equations VIII, and defining “augmented” requirements of means of production *and subsistence*  $\underline{A}_a \equiv A_a + a L_a, \dots, \underline{K}_k \equiv K_k + k L_k$ , one obtains:

$$(\underline{A}_a p_a + \dots + \underline{K}_a p_k)(1+r) = A p_a$$

..... (IX)

$$(\underline{A}_k p_a + \dots + \underline{K}_k p_k)(1+r) = K p_k$$

These equations are equivalent to Sraffa’s equations at the beginning of his chapter 2 (*op.cit.*) and they make what may be called a “Classical system” of value and distribution, or specifically “Ricardo’s system”.

Sraffa assumes (*loc.cit.*) this system to be in a state of “self replacement” with a surplus, which means that  $A \geq \sum \underline{A}_j, \dots, K \geq \sum \underline{K}_i$ , being strictly  $>$  at least once; so there exists a “surplus product”, an excess of the gross product over the

consumption of the means of production together with the goods and services which are necessary for the subsistence of the workers. The system then determines the rate of profit  $r$  and the values of commodities  $\mathbf{a}, \dots, \mathbf{k}$  in terms of some commodity which is taken as a standard, for instance corn (making  $\mathbf{p}_a \equiv 1$ )<sup>14</sup>.

Other Ricardian results can be derived from this formulation by Sraffa, although Sraffa himself has not done it in the book.

From the *givens* in equations VIII one can calculate the Net National Product-NNP  $\{(\mathbf{A}-\Sigma\mathbf{A}_i), \dots, (\mathbf{K}-\Sigma\mathbf{K}_i)\}$ <sup>15</sup>. The “annual labour of the country”,  $\mathbf{L}$ , becomes in the present model the total labour employed in all the industries of the economy, that is to say,  $\mathbf{L} \equiv \Sigma \mathbf{L}_i$  for  $i = \mathbf{a}, \dots, \mathbf{k}$ . One can also define the aggregate wage basket  $\{\mathbf{A}_w, \dots, \mathbf{K}_w\}$ , where  $\mathbf{A}_w \equiv \mathbf{a}N, \dots, \mathbf{K}_w \equiv \mathbf{k}N$ . Being  $\{\mathbf{A}_w, \dots, \mathbf{K}_w\}$  a part (proper subset) of the NNP, the remaining part is the aggregate “surplus product”  $\{(\mathbf{A}-\Sigma\mathbf{A}_i-\mathbf{A}_w), \dots, (\mathbf{K}-\Sigma\mathbf{K}_i-\mathbf{K}_w)\}$ .

Since  $\mathbf{A}_a, \dots, \mathbf{K}_k, \mathbf{L}_a, \dots, \mathbf{L}_k$  are known quantities (for *given* gross outputs in the various industries,  $\mathbf{A}, \dots, \mathbf{K}$ ), one can calculate the quantities of labour which are directly *and indirectly* required per unit of commodities  $\mathbf{a}, \mathbf{b}, \dots, \mathbf{k}$ , being these quantities denoted here by  $\lambda_a, \dots, \lambda_k$ .<sup>16</sup> One can then calculate the quantity of labour which is directly and indirectly required to produce any “composite” commodity, or “basket” of commodities. In particular, the quantity of labour required to produce the NNP is given by:

$$\Lambda \equiv (\mathbf{A}-\Sigma\mathbf{A}_i)\lambda_a + \dots + (\mathbf{K}-\Sigma\mathbf{K}_i)\lambda_k$$

It can easily be shown (see equations in footnote 16) that  $\Lambda = \mathbf{L}$ , i.e.  $\Lambda$  is equal to the “annual labour of the country”, i.e. the aggregate quantity of labour performed in all the industries, during the yearly cycle of production. As in Ricardo’s simple model with “unassisted labour”, this total  $\mathbf{L}$  divides into two

<sup>14</sup> System (IX) may be written, in obvious matrix notation:  $\underline{\mathbf{M}}\mathbf{p}(1+r) = \mathbf{p}$ , where  $\underline{\mathbf{M}}$  is the matrix of “augmented” coefficients of means of production and subsistence (calculated *per unit of product*), and  $\mathbf{p}$  is the vector of prices. Some theorems of Perron-Frobenius, supposing  $\underline{\mathbf{M}}$  to be nonnegative, ensure that  $r > 0$  as well as positive prices. See Pasinetti (1977), Appendix, section 11.

<sup>15</sup> “The national income of a system in a self-replacing state consists of the set of commodities which are left over when from the gross national product we have removed item by item the articles which go to replace the means of production used up in all the industries.” (Sraffa, 1960, p.11)

<sup>16</sup> The following system of linear equations can be solved for  $\lambda_a, \dots, \lambda_k$ :

$$\mathbf{A}_a\lambda_a + \dots + \mathbf{K}_a\lambda_k + \mathbf{L}_a = \mathbf{A}\lambda_a, \dots, \mathbf{A}_k\lambda_a + \dots + \mathbf{K}_k\lambda_k + \mathbf{L}_k = \mathbf{K}\lambda_k$$

parts, namely the “necessary” labour,  $L_N$ , and the “surplus” labour,  $L_S$ , such that  $L_N = \Lambda_N$  e  $L_S = \Lambda_S$ , where:

$$\Lambda_N \equiv A_w \lambda_a + \dots + K_w \lambda_k$$

$$\Lambda_S \equiv (A - \sum A_i - A_w) \lambda_a + \dots + (K - \sum K_i - K_w) \lambda_k$$

The ratio  $L_S/L_N$  is the “degree of exploitation” (Marx), which is equivalent to  $L_N/L$ , or “proportion of the annual labour of the country [that] is devoted to the support of the labourers” (Ricardo), as seen in section 2 above.

As to the relationship between  $L_S/L_N$  and  $r$  in this model, it can be shown that, in equations IX,  $r$  will be lower (or higher) if anyone of the quantities  $\underline{A}_a, \dots, \underline{K}_k$  be larger (or smaller)<sup>17</sup>; which means, in particular, that  $r$  will be lower (or higher) if the *given* wage basket  $\{\mathbf{a}, \mathbf{b}, \dots, \mathbf{k}\}$  have greater (or smaller) quantity(ies) of some component(s), *and not less (or more) of any one (the commodities being the same)*. This is a rather restrictive condition, which was implicit in Ricardo’s versions of the theorem in 1815 and 1817, as seen in sections 1 and 2 above. Under this condition, a “larger” (or “smaller”) basket, in this sense, implies a larger (or smaller) quantity of “necessary labour” and therefore a lower (or higher)  $L_S/L_N$ . There would be, then, in this Ricardian model a positive relationship between  $r$  and  $L_S/L_N$ , though not simply an equality as in equation V. But it does not necessarily follow if some items of the wage basket increase while others decrease, nor does it necessarily hold if the component items are not the same.

#### 4 Sraffa’s basic model

In Ricardo’s model (seen in section 3), Sraffa relax the Classical supposition of a given wage basket required for the subsistence of the worker: “We have up to this point regarded wages as consisting of the necessary subsistence of the workers [or “as consisting of specified necessities

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<sup>17</sup> This result is also an application of one of the theorems of Perron-Frobenius (see Pasinetti, *loc.cit.*), when  $r$  is determined by the production equations of “basic” commodities, including the wage-goods (see Sraffa, 1960, p.7-8), so that matrix  $\underline{M}$  is reduced to an indecomposable matrix (see note 14 above).



If the wage per unit of labour,  $w$ , is expressed in terms of this new standard of value, and being  $w \equiv wL$ , it is the same thing as the relative share of labour in the national income.

Being “profit” defined as the difference between “revenue” and “cost” – be it in a firm, an industry or in social production as a whole – its aggregate value, or the total amount of profits in the economy, is the difference between, on the one hand, the value of the gross product  $\{A, \dots, K\}$  and, on the other hand, the “costs” of materials and productive services together with the costs of labour, that is to say, the sum of the total value of the means of production consumed  $\{(\sum A_i), \dots, (\sum K_i)\}$  with the total wage bill  $w$ . So, the total amount of profits is equal to the value of the NNP (which is the difference between the value of the gross product and the value of the means of production consumed) minus the total wage bill, or  $1 - w$ .

Following Sraffa, one can now suppose that the real wage  $w$  or the profit rate  $r$  is *given* (an independent or exogenous variable), in the interior of feasible intervals<sup>19</sup>. Since  $A, \dots, K; A_a, \dots, K_k; L_a, \dots, L_k$  are also given, then equations X and XI determine the other distributive variable ( $r$  or  $w$ )<sup>20</sup> and the relative prices of all commodities  $p_a, \dots, p_k$ ; and therefore they determine the aggregate amounts of wages and profits, that is, the distribution of national income between labour and capital. It is supposed here that the *given*  $w$  or  $r$  are such that  $0 < w < 1$ , so that both the share of profits and the share of wages are positive.

## 5 A Sraffian development of the theory of exploitation

Sraffa’s reformulation of Ricardo’s system, considered in the previous section, applies to a situation in which the real wage can exceed the requirements of subsistence. It can incorporate a concept of exploitation which is in turn a development of a concept used by Marx in describing a similar situation, in which the wage basket includes more than strict necessities, and

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<sup>19</sup> “The rate of profits, as a ratio, has a significance which is independent of any prices, and can well be ‘given’ before the prices are fixed. It is accordingly susceptible of being determined from outside the system of production, in particular by the level of the money rates of interest.” (Sraffa, 1960, p.33)

<sup>20</sup> There is in this model an inverse relationship between  $r$  and  $w$ , as shown by Sraffa (1960, p.40).

therefore the labour required to produce it is more than “necessary”. Accordingly, in describing such a situation, Marx refers to “exploitation” as extraction of “unpaid labour”, instead of “surplus labour”:

“[in a process of capital accumulation in the economy as a whole] the requirements of accumulating capital may exceed the increase of labour-power or the number of labourers; the demand for labourers may exceed the supply, and, therefore, wages may rise. ...[the labourers] can extend the circle of their enjoyments; can make some additions to their consumption-fund of clothes, furniture, etc., and can lay by small reserve-funds of money. But just as little as better clothing, food, and treatment, and a larger peculium, do away with the *exploitation* of the slave, so little do they set aside that of the wage-worker. ...Wages, as we have seen, by their very *nature*, always imply the performance of a certain quantity of *unpaid labour* on the part of the labourer ... such an increase [of wages] only means at best a quantitative diminution of the unpaid labour that the worker has to supply.” (Marx, 1867, p.619)

In his basic formulation, in which the wage (or “price of the labour-power”) is equal to the “value” of the labour-power, “paid labour” is treated as equal to “necessary labour”, and “unpaid labour” as equal to “surplus labour”<sup>21</sup>. This is a particular case of a more general formulation, which is evident in the passage just quoted, where “unpaid labour” takes the place of “surplus labour”; and their magnitudes are different, which affects the *degree* of exploitation, but not its substance, namely “unpaid labour”.

This change could be incorporated into Sraffa’s version of Ricardo’s model (section 3 above), where the aggregate wage basket  $\{A_w, \dots, K_w\}$  might be redefined so that it may exceed subsistence requirements, and thus include a share of the *surplus* product. “Exploitation” would accordingly be redefined in terms of “paid labour” and “unpaid labour”, instead of “necessary labour” and “surplus labour”; being “paid labour” the part of  $L$  equal to  $A_w \lambda_a + \dots + K_w \lambda_k$  and “unpaid labour” the remaining part. One might suppose that wages are *entirely* spent for the purchase of some of the commodities  $a, \dots, k$ ; so that:

$$A_w p_a + \dots + K_w p_k = wL$$

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<sup>21</sup> See Marx(1867), cap.XVIII *et passim*.

Even if the wage basket  $\{A_w, \dots, K_w\}$  remains indeterminate, the assumption (made in section 3 above) that it is a part (proper subset) of the NNP implies the existence of exploitation, in the sense of obtaining “unpaid labour”.

However, “paid labour” will not be defined here in terms of the quantity of labour required (directly and indirectly) to produce the basket of commodities which is *obtained* in *expending* the wages; nor in terms of the basket which is *consumed* by the workers.

In the theory of Classical Political Economy about capitalism, including Ricardo’s theories reviewed above, the wage was treated as consisting of a necessary basket of means of subsistence. But, it was assumed, what the workers receive as *payment* is an amount of *money*, or *general purchasing power*, or *command* over commodities *in general*, which can be expressed in terms of some particular commodity, e.g. wheat or steel, or in terms of some composite commodity, e.g. {1t. Wheat, 2t. Steel}. By means of the wages in money, labour *can buy* an infinite variety of commodities (simple or composite). Each one of these (with precise quantity(ies) of one or more products) requires directly and indirectly for its production a definite quantity of labour, that is to say, this required quantity of labour depends on the commodity in terms of which is expressed the *purchasing power* of the wage, or the exchange value of labour. A convenient *and meaningful* standard of value, for expressing the *aggregate* amount of wages *paid*, or the total exchange value of the “annual labour of the country” ( $L$ ), is the Net National Product-NNP. In effect, being the aggregate amount of wages,  $w$ , expressed in terms of the NNP, it is *ipso facto* the relative share of labour (of all workers) in the Net National Product, which is the annual *result* of social production. But  $L$ , by means of  $w$ , *commands* (is *equivalent* to) a “share” of the NNP, which is here defined as a part of NNP containing equal fractions of all its components (quantities of goods and services). Now, this “share” of the NNP requires directly and indirectly for its production a definite quantity of labour, i.e. a part of the total labour required to produce the whole NNP, this total labour being  $\Lambda$ , which is in turn equal to  $L$  (as seen in section 3 above)<sup>22</sup>. The part of the “annual labour of the country” ( $L$ )

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<sup>22</sup> To say that  $L$  “commands” a “share” of the NNP is equivalent to saying that this share commands  $L$ , in the same way that the *whole* NNP commands a definite quantity of labour, which is proportionately

equal to such quantity of labour, which is required to produce the “share” of the NNP *commanded* by  $w$ , is here defined as “paid labour” of the set of all workers; and the remaining part is their “unpaid labour”. The latter is in turn equal to the quantity of labour required to produce the remaining “share” of the NNP, which is the part *commanded* by the aggregate amount of profits.

One can now resume the Marxian definition of “exploitation” as extraction of “unpaid labour”, and also his definition of “degree of exploitation” as the ratio between “unpaid labour” and “paid labour”, although these variables have been redefined here. Denoting the degree of exploitation by  $x$ , “paid labour” by  $L_P$ , and “unpaid labour” by  $L_{UP}$ , then  $x \equiv L_{UP}/L_P$ , which is conveniently equal to the ratio between the amounts of profits and wages in the national income; and, due to the choice of units, this equality takes the simple form:

$$x = (1 - w) / w \quad (XII)$$

As seen in the previous section, the rate of real wages  $w$  (in terms of the NNP) or the rate of profit  $r$  may be supposed to be *given*, in the *interior* of feasible intervals.  $A, \dots, K$ ;  $A_a, \dots, K_k$ ; and  $L_a, \dots, L_k$  being also given, then the system of Ricardo-Sraffa (equations X and XI) determines the other distributive variable ( $r$  or  $w$ ) and the relative prices of all commodities. The relative shares of labour and capital in the national income,  $w$  and  $1 - w$ , are thereby given or determined; and the degree of exploitation,  $x$ , is then determined as a *result*, according to equation XII<sup>23</sup>.

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larger than  $L$ , and therefore larger than  $\Lambda$ . Cf. Smith: “As in a civilized country there are but few commodities of which the exchangeable value arises from labour only, rent [of land] and profit contributing largely to that of the far greater part of them, so the annual produce of its labour [the product of the “annual labour of the nation”] will always be sufficient to purchase or command a much greater quantity of labour than what was employed in raising, preparing, and bringing that produce to market.” (Smith, 1776, vol. I, p. 71)

<sup>23</sup> It is evident that, if  $w$  is increased (or diminished) independently, then  $x$  falls (or rises). But there is in this model an inverse relationship between  $r$  and  $w$  (see Sraffa, 1960, p. 40) and so there results a positive relationship between  $x$  and  $r$ . Alternatively, if  $r$  is the independent variable,  $x$  rises (or falls) in consequence of an increase (or diminution) of  $r$ , through the same inverse relationship between  $r$  and  $w$ .



## 6 Comments, possible extensions, conclusion

It is worth recalling here Ricardo's proposition:

"It is according to the division of the whole produce of the land and labour of the country, between the three classes of landlords, capitalists, and labourers, that we are to judge of rent [of land], profit, and wages... *It is not by the absolute quantity of produce obtained by either class, that we can correctly judge of the rate of profit, rent, and wages, but by the quantity of labour required to obtain that produce.*" (Ricardo, 1817, p.64; emphasis added)

The reformulation of Ricardo's theory about exploitation that has been suggested here implies substituting, in the passage just quoted, the word "obtained" by the word "commanded" (in the sense of *purchasing power*). The resulting theory is simpler than Ricardo's as it does not involve the determination of some basket of commodities *obtained* or *consumed* by the class of workers<sup>24</sup>. The distribution of the national product is described in its aspect of *payment* or *appropriation* of income, rather than the *utilization* or *expenditure* of income, being thereby associated to the division of the "annual labour of the country" between "paid" and "unpaid" labour. Here, "paid labour" and "unpaid labour" are modified versions of those variables defined by Marx in one of his concepts of exploitation, or as one of the aspects of his concept of exploitation.

In developing further the suggested formulation, one can relax the simplifying assumption that society is partitioned into a class of "workers" (individuals who belong only to this class) and other classes like "capitalists", "landlords" etc. (individuals who do not belong to the class of workers), as in the passage by Ricardo just quoted. It is admitted that anyone who work for wages may also earn other kinds of income, like profits, interest, rents from properties

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<sup>24</sup> Such determination belongs to spheres of the economy other than *production*, namely *expenditure* or *consumption*, being logically separable from the *wage relation* (rent, or payment for the use, of labour-power), a "social relation of production" (social relation through which production takes place; in this case, the performance of labour). Besides, the worker may obtain income from other sources, and also may save part of his total income. In the passage by Marx quoted at the beginning of section 5 above, the worker who receives higher wages "can lay by small reserve-funds of money". This fund, or part of it, may even be converted into capital (being invested in company shares, for instance) which implies "self-exploitation" (under the aggregate concept which has been suggested here), if the person continues to work for wages as an employee.

etc.<sup>25</sup>, although it may happen that a sub-set of individuals, or even the far greater part of them, be only “workers”, or “mainly” workers (in particular, in the sense that more than half of their income be obtained from wage labour).

It is worth reminding that Marx described the exploitation of workers by capitalists as being also exploitation of labour (or rather labour-power) by capital, sometimes conceiving it in aggregate form, as Ricardo had done and has also been done here:

“The rate of exploitation is therefore an exact expression for the degree of exploitation of labour-power by capital, or of the labourer by the capitalist.” (Marx,1867,p.218)

“[the average profit (at the general rate) of the individual capitalist, or of all the capitalists in each individual sphere of production] is due to the aggregate exploitation of labour on the part of the total social capital, i.e., by all his capitalist colleagues” (Marx,1894,p.170)

The development of Ricardo’s theory about exploitation, here submitted, has been formulated under other simplifying assumptions, such as homogeneous labour, no joint production, only circulating capital, free land, uniform rates of profit etc. The relaxation of these suppositions – admitting heterogeneous labour, joint production, fixed capital, rent of land, monopoly rent (determined, for example, through “mark up” pricing) and other generalizations or extensions<sup>26</sup> - would present no great difficulties, it seems. In particular, the inclusion of joint production, fixed capital and rent of land could proceed along the lines of Sraffa(1960), Part II.

The characteristic proposition – a theorem of *existence* of “exploitation” – will obtain provided there be an annual Net National Product-NNP which require

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<sup>25</sup> “[the price of] all the commodities which compose the whole annual produce of the labour of every country, taken complexly, must resolve itself into the same three parts [rent of land, wages of labour and profits on capital], and be parcelled out among different inhabitants of the country, either as the wages of their labour, the profits of their stock, or the rent of their land. ...When those three different sorts of revenue belong to different persons, they are readily distinguished; but when they belong to the same they are sometimes confounded with one another, at least in common language.” (Smith,1776,vol.I,p.69,70)

<sup>26</sup> The argument presented here defines the wage as per unit of labour (as exchange value of labour itself), but it can incorporate the distinction, made by Marx, between labour and labour-power, treating the wage as rental price of the labour-power; so that “it is no longer possible to be misled, by the formula Unpaid labour/Paid labour, into concluding, that the capitalist pays for labour and not for labour-power” (Marx,1867,p.534). In effect, the argument supposes the duration (length of time) and intensity of labour to be *given*, and thus the wage per worker (employed for one year) can be translated into the wage per unit of labour, and vice versa, by means of the following identity:  $L/N \equiv (L/H)(H/N)$  where  $H/N$  is the average working time (e.g. in hours) per worker and  $L/H$  is the average intensity of work (labour performed per unit of time).

directly and indirectly for its production a definite quantity of labour equal to the “annual labour of the country”<sup>27</sup>; and provided that the value of the NNP be distributed between wages of labour and other classes of income, i.e. classes of *purchasing power* over parts of the NNP. In particular, the aggregate amount of wages *paid* can always be expressed in terms of the NNP, *commanding* a fraction or “share” of it (with the same composition, as defined in section 5 above), for the production of which it will be necessary the same fraction of the quantity of labour required directly and indirectly to produce the whole NNP. To this fraction will be equal the “paid” fraction of the “annual labour of the country”. The remaining “unpaid” fraction will be positive; in other words, there will be “exploitation” in the sense of *obtaining “unpaid labour”*<sup>28</sup>. Thus one can say that *earning profits, interest, rent and other forms of non-labour income*, thereby obtaining the command over “shares” of the NNP, implies *obtaining “unpaid labour”*.

We have after all the same picture of the distribution of national income, between wages and other incomes, as that conveyed by Ricardo’s “corn-ratio” theory of profits. As in the production of corn, which yields a net product (including the share *commanded* by labour), so the “shares” of the NNP that are *commanded* by labour, capital, land etc. consist of the same commodity as the NNP, though in different quantities; and such “shares” are therefore proportional to the quantities of labour required to produce them, implying a proportional division of the quantity of labour directly employed in the production of the whole NNP. Thus Ricardo’s “corn-ratio” is a kind of “corn parable” for the distribution of the national income<sup>29</sup>.

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<sup>27</sup> If there are  $n$  kinds of labour: for each one of them let  $L_j$  be the quantity employed in the yearly cycle of production, being  $j=1,2,\dots,n$ . The quantity of labour  $j$  required directly and indirectly to produce the *Gross Product* (GP) of the economy is the sum of the direct plus the indirect quantities; being the latter the quantity of labour  $j$  required directly and indirectly to produce the necessary Means of Production (MP):  $\Lambda_{jGP} = L_j + \Lambda_{jMP}$ . But the set of commodities making the Gross Product can be *partitioned* into two sub-sets: the replacement of the means of production and the Net Product (NP), so that:  $\Lambda_{jGP} = \Lambda_{jMP} + \Lambda_{jNP}$ . Therefore:  $\Lambda_{jNP} = L_j$ .

<sup>28</sup> If there are various kinds of labour (see previous note) a “share” of the NNP requires the same fraction of the quantities of all kinds of labour that are required to produce the whole NNP. Accordingly, all kinds of “annual labour of the country” split into the same fractions of “paid labour” and “unpaid labour”, and thus the *aggregate* degree of exploitation is equal to the common ratio of “unpaid labour” to “paid labour”.

<sup>29</sup> “Many years later [after 1815], an echo of the old corn-ratio theory (which rendered distribution independent of value) can perhaps be recognised when Ricardo in a moment of discouragement with the difficulties of value writes do McCulloch: ‘After all, the great questions of Rent, Wages, and Profits must be explained by the proportions in which the whole produce is divided between landlords, capitalists, and

## References

CATEPHORES, G. (1989) *An introduction to Marxist economics*. London: Macmillan, 1989.

DMITRIEV, V.K. (1898) La teoría del valor de David Ricardo: un intento de análisis riguroso. In: \_\_\_\_\_. *Ensayos económicos sobre el valor, la competencia y la utilidad*. México: Siglo XXI, 1977.

MACEDO, L.A.M. (1990) "The formation of Ricardo's early theory of profits: a reconstruction based on Smith, Malthus and Horner". Unpublished Ph.D. thesis, University College London, 1990.

\_\_\_\_\_. (1991) A teoria 'agrícola' de lucros de Ricardo derivada da teoria de renda fundiária de Smith. *Nova Economia*, v.2,n.2:161-173, 1991.

MARX, K. (1867) *Capital*. Book I. New York: International Publishers, 1967. Vol. I.

\_\_\_\_\_. (1894) *Capital*. Book III. New York: International Publishers, 1967. Vol. III.

PASINETTI, L.L. (1977) *Lectures on the theory of production*. New York: Columbia University Press, 1977.

RICARDO, D. (1815) An essay on the influence of a low price of corn on the profits of stock.. In: RICARDO, D. *The works and correspondence of David Ricardo*. Vol. IV. Cambridge: University Press, 1951.

\_\_\_\_\_. (1817) Principles of Political Economy and taxation. In: RICARDO, D. *Works...* Vol. I. Cambridge: University Press, 1951.

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labourers, and which are not essentially connected with the doctrine of value' [letter of 13 June 1820]." (Sraffa, 1951, p. xxxiii)

SAMUELSON, P. (1974) Insight and detour in the theory of exploitation: a reply to Baumol. *Journal of Economic Literature*, 12(1): 62-70, March 1974.

SMITH, A. (1776) *An inquiry into the nature and causes of the wealth of nations*. Oxford:University Press, 1976. [Reprint by LibertyPress / LibertyClassics, Indianapolis,1981] Vol. I.

SRAFFA, P. (1951) Introduction [to Ricardo's *Principles*] In: RICARDO, D. *Works...* v. I. Cambridge: University Press, 1951.  
\_\_\_\_\_. (1960) *Production of commodities by means of commodities*. Cambridge: University Press, 1960.

STEEDMAN, I. (1977) *Marx after Sraffa*. NLB/Verso Ed.: London, 1981.