Evaluating Marginal Productivity: An Exploratory Paper using Data Envelopment Analysis

By

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Abstract

Data envelopment analysis (DEA) is a linear programming technique that can be used to evaluate and rank or benchmark the performance of different businesses, non-profit organizations, public sector agencies, and even national economies. DEA can rank order these or any set of entities called decision-making units or DMUs according to how well they minimize inputs in order to maximize output(s), or, in other words, rank order the DMUs according to how efficient they are. This paper uses DEA to rank order 16 OECD nations according to how efficiently they attained their national output levels through capital and labor inputs minimization during the decade of the 2000s. Next, these efficiency scores as well as other measures of the income shares or returns to labor and capital for each economy are used in order to see if there is some type of support for an aggregate, economy wide productivity theory of distribution. Other variables (exploitation measures, trade union density, human capital levels, etc.) are also employed to see if alternative explanations of labor and capital income shares have some merit. Although different measures of marginal productivity have

been used at aggregate levels to one degree or another to predict payments to factor inputs in order to assess marginal and average productivity theory, the research done for this paper has not found any other works that have used DEA, and so this paper seeks to make a new and perhaps unique empirical contribution to the literature on aggregate marginal and average productivity theories of labor and capital income distribution.

JEL Codes:

E25 - Aggregate Factor Income Distribution

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Introduction

The concepts of diminishing returns, and average and marginal productivities in microeconomics have been debated by both heterodox and neoclassical economists (Keen 2011). It is when these concepts, especially the concept of the marginal productivity of capital, are used by macroeconomists to describe income shares at aggregate levels, that the debate becomes even more contentious. According to Blaug (1975), Moseley (2012), and Cohen and Harcourt (2003), the use of marginal productivity to explain factor rewards in a macroeconomy comes more or less from an attempt by mainstream economists to respond to Marx's claim that profits or rewards to capital come from the exploitation of workers and from the capitalist taking the surplus value of the workers' efforts. Using a production function where Q = f (K, L), essentially and simply stated, marginal productivity of capital and labor puts forth that

$$\Delta Q / \Delta K = r \qquad (1)$$

And

$$\Delta Q / \Delta L = w \qquad (2)$$

where Q = output, K = capital, L = labor, w= wage rate, and r = rate of interest (Branson 1989, Jones 2002, and Romer 2012). These concepts lead to the concepts of downward sloping demand curves for labor and capital (Cohen and Harcourt 2003), and according to Cobb and Douglas (1928) and Solow (1956, 1957) using a Cobb-Douglas production function and applying regression, aggregate labor and capital estimates "fit" very well the aggregate factors shares or payments over several decades. The shares are fairly constant—in the US, around one third of national income goes to capital, and approximately two-thirds to seventy percent of it goes to labor (Jones 2002, page 14).

Later, Robinson (1953-54), Sraffa (1960) and others (Cohen and Harcourt 2003, Garegnani 2012) challenged the theory of the marginal productivity of capital on the grounds that the theory involved circular reasoning (i.e., the rate of interest determines how much capital is demanded, yet the amount of capital used in production determines the rate of interest) and that an aggregate capital measure cannot exist because capital is not homogenous (Keen 2011). Also, as interest rates fall, sometimes a production technique can use less capital rather than more due to "re-switching", although marginal productivity of capital logic would dictate a downward sloping demand curve for capital with greater amounts of capital demanded as rates fall (Sraffa 1960, Samuelson 1973 pages 611-616, Rocangalia 1978, Grieve 2012). Next, the strong regression fits that result from using a Cobb-Douglas function are due to the fact that the model is basically using data that is part of an accounting identity. GDP, or Q, has to equal rK + wL due to national income accounting definitions and standards, and hence using capital and labor values to predict levels of output should result in very good regression fits (Shaikh 1974, Simon, 1979, Lavoie 2008). Next, according to critics, what really is important is the fact that capital earnings come from the power that capitalists have over the means of production which enables them to extract a surplus from labor for profit and reinvestment—the *a priori* distribution of wealth and income as well as property rights determine factor payments or prices (wages and interest rates) (Dobb 1973, Cohen and Harcourt 2003). The fact that there really is no "supply of capital" theory is another problem noted by others (Moseley 2012). Also, the use of general equilibrium theory is also deemed to be an inadequate way to explain "r", because the results of general equilibrium analysis are not stable and do not always tend toward equilibrium and rely upon fixed coefficients (Brue 1994, Cohen and Harcourt 2003). Additionally, the recent reliance on the use of capital growth theory has been employed to explain the accumulation of capital and/or capital per worker in an economy, and these models employee some type of marginal productivity of capital and

labor concepts as well as a savings preference for the "typical" household as foundations (Romer 2012, Jones 2002 pp. 22-24). Finally, the assumption that aggregate production has to be done according to constant returns to scale in order to arrive at factor shares is deemed overly simplistic and unrealistic (Cohen and Harcourt 2003, Alam 2013).

In addition to using a Cobb-Douglas production function to assess the productivity of capital and labor, one could use other techniques. Over a given time period, the change in national output over the change in the value of capital stock (fixed assets) of a nation or the change in national output over the annual consumption of fixed capital could be used as a proxy for the marginal productivity or the marginal revenue product of capital. Neoclassical theory indicates that these values should equal an interest rate, a return on investment estimate, or the percentage of GDP that is gross or net operating surplus for each nation for each corresponding year. The last item is the one most commonly used since it is the one that a Cobb-Douglas model predicts so well according to some of the sources mentioned above. Putting "r" in terms of national income shares (gross or net operating surplus) is the usual way of expressing factor payments in most of the literature reviewed (Jones 2002, Romer 2012 among others).

Likewise, a simple marginal product of labor or marginal revenue product of labor (MPL) could be calculated by taking the change in GDP or output and dividing it by the change in labor pay from year to year (or change in hours, etc.). Neoclassical beliefs suggest that this should be equal to a wage rate (hourly or annual) or labor's share of national income as a portion of GDP.

Average labor and capital productivity can be estimated several ways. For capital, one way is by dividing GDP for each nation by fixed capital consumption, or by total fixed assets (if such estimates exist). For labor, GDP could be divided by labor hours, total labor pay, etc.

Finally, productivity can be assessed by using a technique called data envelopment analysis (DEA). DEA is based on linear programming, and is a non-parametric technique that ranks different entities (called decision making units, or DMUs) according to how efficiently they combine inputs to attain their respective levels of output(s). Since it is non-parametric, no *a priori* assumptions about production techniques (Cobb-Douglas, linear, fixed factors, etc.) are necessary. From a list of the DMUs' inputs and outputs, DEA creates a production possibility frontier (PPF) that approximates the minimum use of inputs or resources to get a maximum output level. Each DMU is assigned an efficiency score based on how close it is to the PPF, and those DMUs which match PPF efficiency or have a combination of inputs and outputs which would place them on the PPF receive a score of 1.0. Those DMUs which score below 1.0 fall below PPF efficiency and are deemed "inefficient".[1]

DEA has been used in the past to measure the performance of different nations with regard to efficiency, whether efficiency in labor productivity (Maudos, Pastor and Serrano 2000), in the delivery of social services (Golany and Thore 1997), or efficiency in GDP production (Lambert 2011). It has been proposed as a useful tool for economists because of its ability to highlight "x-inefficiency" in production or any economic system (Leibenstein and Maital 1992).

DEA can also estimate whether a DMU is experiencing decreasing, constant, or increasing returns to scale with regard to output production, although traditional marginal productivity theory assumes an economy where "constant returns to production" and competitive markets exist, assumptions which some call parts of the "neo-classical fable" or a set of parables (Samuelson 1973).

[1] Admittedly this is a narrow definition of efficiency. Yet, it is in the spirit of the neoclassical point of view that agents and markets seek to maximize profits and/or output through minimizing costs and/or inputs, and this paper tries to give the neoclassical point of view the opportunity to show some validity since it has come under heavy criticism in the past . Hence, this narrow definition is used. Heterodox economists would probably disagree with such a narrow definition, or even that efficiency as a concept exists. For example, Simon (1947) argued that managers in organizations act in a way such that their behavior often is not oriented toward maximization but instead toward "satisficing" or obtaining satisfactory results.

Therefore, this paper will mostly focus on evaluating constant returns to scale (CRS) production with regard to support for neoclassical theory, although variable returns to scale (VRS) scores are used in the analyses done in this paper as well. VRS scores are offered as a point of contrast.

A DEA score represents an elasticity which measures "the relative change in output compared to the relative change in input" according to Cooper, Seiford, and Tone (2006, pp. 119-121). Therefore, a DEA score is a way of assessing productivity just as the other measurements mentioned so far.[2]

Although one purpose of this paper is to test hypotheses explaining factor shares by using marginal productivity concepts as well as DEA, other variables that could explain factor income shares, such as labor exploitation, human capital, property rights, etc., are also examined and used in various models. These are added because the productivity measures do not come close to explaining all the variation in factor rewards and are used to test heterodox hypotheses of factor rewards.

This paper proceeds as follows. The next section (Methods) outlines the variables and methods used to evaluate different forms of national economic productivity for labor and capital for different OECD nations during the years 2000 to 2010. After this section, a results section summarizes the main findings of the models developed and used to estimate various measurements of "w" and "r". Finally, a discussion and conclusion section reviews the major findings in light of their implications for economic theory and pedagogy.

[2] If an efficiency score, E, can be expressed similar to an elasticity as $E = dQ/dI * I/Q$, where Q is output and I is input, then the marginal product of the input can be found by multiplying both sides of the equation by Q/I. In this paper, since several measures of capital and labor are used in the DEA of each factor of production, a derivative is conceptually possible, but it is not like the simple one just mentioned where dQ/dI has just one input. One could multiply E by Q / (Sum of Inputs), but in the course of writing this paper, it could not be verified if doing so would be an accurate way to get at a productivity differential. When this calculation is done for labor and capital scores (both CRS and VRS scores), the adjusted E for capital is not a good predictor (not statistically significant) and the adjusted E for labor is significant only at $p < 0.10$. Therefore, the efficiency scores (an elasticity) are used as proxies for some type of marginal productivity.

Methods

(Insert Table 1 around here)

Data from 15 (for labor analysis) and 16 (for capital analysis) OECD member nations[3] (http://stats.oecd.org/Index.aspx?#) as shown in Table 1 from the period of 2000 to 2010 (all data are in US dollars and adjusted for purchasing-power-parity) were analyzed in two steps:

1. DEA

Efficiency scores for each nation were calculated for labor and capital. Labor's productivity was assessed using total employment in thousands and total hours worked in thousands as inputs and GDP in thousands of dollars as an output. Capital productivity was measured by using total fixed assets in thousands, consumption of fixed capital, and net fixed assets formation/additions in thousands of dollars as inputs and GDP in thousands of dollars as an output.

Although Cobb-Douglas models assess the productivity of labor and capital together in equation form, in DEA any efficiency score is for all inputs used, and it would be hard to discern the contributions of each input, capital and labor, individually. Therefore, in this paper labor and capital are assessed separately in order to pinpoint their efficiency in producing output.

Percentages, per capita estimates, and ratios were not used in the data envelopment analysis since it is a technique which does not assume a priori relationships among the inputs and outputs, and the avoidance of using ratios and instead using absolute amounts is considered standard practice unless the Banker, Charnes, and Cooper (BCC) formulation of DEA is used for analysis (Hollingsworth and

[3] Only 15 and 16 out of 35 or so member nations had consistent data for labor and capital (or fixed assets) during this time period. Slovenia is missing from Table 1 in the analysis of labor because its total employment in thousands was not available and could not be pinpointed from a reliable source, and only 16 nations had fixed assets data on the OECD website.

Smith 2003). When used, with or without the ratio data, the BCC method is very good for capturing increasing and decreasing returns to scale in production whereas the Charnes, Cooper and Rhodes method is good for measuring constant returns to scale efficiency (Cooper, Seiford, and Tone 2006).

The DEA model can be stated as (Charnes et al, 1978; Fare et al., 1994; Nolan et al., 2001):

$$\text{Maximize efficiency score } (jp) = \frac{\sum_{r=1}^{t} u_r y_{rjp}}{\sum_{i=1}^{m} v_i x_{ijp}} \qquad (3)$$

$$\text{Subject to} \quad \frac{\sum_{r=1}^{t} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \leq 1, \quad j = 1, \ldots, n, \qquad (4)$$

$$u_r, v_i \geq \varepsilon, \qquad \forall r \text{ and } i, \qquad (5)$$

$$u_r, v_i \geq \varepsilon, \qquad \forall r \text{ and } i, \qquad (6)$$

where

y_{rj} = amount of output r produced by DMU j,

x_{ij} = amount of input i used by DMU j,

u_r = the weight given to output r,

v_i = the weight given to input i,

n = the number of DMUs,

t = the number of outputs,

m = the number of inputs,

ε = a small positive number.

To ease computational complexity associated with the fractional nonlinear form of the above equations, the above equations can be converted into a linear program as follows.

Maximize efficiency score (jp) = $\sum_{r=1}^{t} u_r y_{rjp}$ (7)

Subject to $\sum_{i=1}^{m} v_i x_{ijp} = a,$ (8)

$$\sum_{r=1}^{t} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0, \qquad j=1,\ldots,n, \tag{9}$$

$$-u_r \le -\varepsilon, \qquad r=1,\ldots,t, \tag{10}$$

$$-v_i \le -\varepsilon, \qquad i=1,\ldots,m, \tag{11}$$

where a = an arbitrarily set constant (e.g., 1 or 100).

By solving the above equations (7)-(11), the efficiency of DMU (jp) is maximized subject to the efficiencies of all DMUs in the set with an upper bound of 1. The above model is solved n times to evaluate the relative efficiency of each DMU. Notice that the weights u_r and v_i are treated as unknown variables whose values will be optimally determined by maximizing the efficiency of the targeted DMU jp. An efficiency score (jp) of 1 indicates that the DMU under consideration is efficient relative to other DMUs, while an efficiency score of less than 1 indicates the DMU under consideration is inefficient. In a broader sense, an efficiency score represents an economy's ability to transform a set of inputs (given resources) into a set of outputs. The above model also identifies a peer

group (efficient DMUs with the same weights) for any inefficient DMU (Boussofiane, *et al*, 1991, Anderson, *et al*, 1999). The results of the 2010 DEA of the 15 and 16 nations is presented in the appendix as an example.[4]

2. Least squares regression

The efficiency scores generated from the DEA analysis were used as independent variables and used with the others listed below to predict the following dependent variables:[5]

Dependent Variables:

a. Net Operating Surplus as a Percentage of a Nation's Fixed Assets or Capital Stock (NOS / Fixed Assets Pct.). This is used a proxy for "r" or a return on a capital. NOS is Gross Operating Surplus less capital consumption allowances and less the net of business subsidies and taxes. Fixed assets include assets lasting over more than one year, including machinery, tools, commercial property, copyrights, patents, and intellectual property.[6] Other estimates for "r" could include NOS as a percentage of GDP, long term interest rates, and long term interest rates net of inflation, yet using these variables as dependent variables yield models with serial correlation. Despite attempts to re-specify the models using all the independent variables listed below, there was no way to avoid the serial correlation. Therefore,

[4] The DEA scores for each nation for each year (2000 to 2010) is available from the author upon request.
[5] Descriptive statistics for all variables can be provided by the author upon request.
[6] More specifically, according to OECD:
Fixed assets are defined in national accounts as non-financial produced assets that are used repeatedly or continuously in production for more than one year. Fixed assets include not only dwellings, buildings, structures, machinery and equipment but also cultivated assets such as livestock for breeding and vineyards. They also include intangible assets such as computer software and entertainment, literary or artistic originals. http://stats.oecd.org/glossary/detail.asp?ID=998

this variable is used as a proxy for "r", the payment to capital. (Data dource: http://stats.oecd.org/Index.aspx?#).

b. Total Wages and Salaries as a Percentage of GDP (Labor Income Share). This is used as a proxy for "w", and is commonly cited as a constant proportion over time when the Cobb-Douglas production function is employed to predict factor incomes shares. Average pay per hour and annual average pay per year were also tried as dependent variables, but the models suffered from serial correlation despite attempts to re-specify. Therefore, this variable is used as the best proxy for "w" or labor factor shares/payments. (Source: http://stats.oecd.org/Index.aspx?#).

Independent variables:

a. Index of Economic Freedom by the Heritage Foundation (Overall Score). Since property rights and the freedom to engage in business activities with as little government interference as possible are important to profits in a capitalistic economy, this variable, an index published for different nations published by the conservative Heritage Foundation (http://www.heritage.org/index/) is used as a predictor. The hypothesis is that the higher the index is (a higher score is favorable toward property rights and *laissez faire* practices), the higher the capital returns should be in an economy whereas labor returns should be lower, all else held constant. This paper uses the overall Heritage Index rating/score which rates nations according to property rights, taxation, size of government, degree of market regulation, etc.

b. Inflation rate. Standard economic theory predicts that, holding everything else constant, a greater inflation rate discourages investment and adversely impacts business profitability (rising expenses) and also adversely impacts wages and salaries (purchasing power is less), so the hypothesis is that the greater the inflation rate, the lower returns to capital and labor. (Source: http://stats.oecd.org/Index.aspx?#).

c. The percentage of a nation's labor force that is comprised of college graduates (BA/BS degree) or higher (College grads). The hypothesis is that the more educated a workforce is, the greater the returns to labor (wages and salaries are higher) and the lower the returns to capital (the surplus is lower) on average, all else held constant. This is traditional, mainstream economic reasoning, although an excess of college grads could show an opposite effect. (Source: http://stats.oecd.org/Index.aspx?#).

d. The nation's unemployment rate (Unemp. Rate). All else held constant, the greater/lower the unemployment rate in a nation, the lower/higher the wages should be. As to a rate of return, the hypothesized impact that unemployment has on it is contingent upon 1) how much unemployment exists (if too high, profits and returns are low because consumer demand should be down) and 2) whether the unemployment mitigates wage increases or even causes wage declines. In the first case, somewhat of a Keynesian view, one would expect profits and capital returns to be down compared to better time periods, whereas in the second case, one would expect profits to be normal or even increase if labor is being exploited enough to

make up for lost profits, which is a somewhat Marxian view. (Source: http://stats.oecd.org/Index.aspx?#).

e. Trade Union Density. This is the percentage of the nation's labor force that is unionized. Trade unions have been cited as gaining wage and benefit concessions for workers and increasing the living standards of members in different nations. (Alderson and Nielsen 2002, Crepaz and Birchfield 2002, Minnich 2003, and Lambert 2011). (Source: http://stats.oecd.org/Index.aspx?#).

f. Profit to Wage Ratio. This is the percentage of each nation's NOS as a portion of all wages and salaries within each nation. This is used as a Marxian concept of exploitation where the logic of capitalism dictates that owners and managers try to maximize profits at the expense of workers' wages (Amudsen 1981, Kalmans 1997, Lambert 2012). The higher the value of this variable is, the lower wages are and the higher NOS should be, all else held constant. It is presented as an alternative to neoclassical economic theory regarding the productivity of labor and capital.[7] (Source: http://stats.oecd.org/Index.aspx?#).

g. Average Annual Hours Worked per Worker/Employee (Average annual hours worked). This is used as another measurement of the Marxian concept of exploitation and is offered as another alternative to neoclassical economic theory. Longer work hours for part and full-time employees (entrepreneurs and the self-employed are not included) are associated with greater labor exploitation and higher surplus values in traditional Marxist literature if real compensation is falling and

[7] This variable could also be considered a rate of exploitation variable where surplus is divided by payments to labor, although no distinction would be made between "productive" and "non-productive" labor as is often done in Marxian economics (Baran and Sweezy 1966, Shaikh and Tonak 1994, among many others).

workers are trying to sustain traditional income levels by working more hours. Therefore, the hypothesis is that the higher the average annual hours worked per worker, the lower labor income shares and the higher NOS are expected to be, everything else held constant. (Source: http://stats.oecd.org/Index.aspx?#).

h. Marginal product of labor or marginal revenue product of labor (MPL). This is one of the key independent variables of interest since this paper is looking for empirical evidence of neoclassical theory and is used to predict the labor income share for each nation. This variable is measured by taking the change in GDP per year for each nation and dividing it by the change in wages and salaries per year for each nation ($\Delta Q / \Delta L$ on an annual basis). If neoclassical theory is correct, this should have some correlation with labor returns, and so the hypothesis is that higher values of this are associated with higher values of labor rewards.

i. Marginal product of capital (MPK). This is another one of the key independent variables of interest because this paper is looking for empirical evidence of neoclassical theory. It is used to predict the NOS/Fixed Assets (or capital) percentage for each nation. This variable is measured by taking the change in GDP per year for each nation and dividing it by the change in the amount of each nation's estimated fixed capital per year ($\Delta Q / \Delta K$ on an annual basis). Similar to labor returns, if neoclassical theory is correct, MPK should have some correlation with capital returns, and so the hypothesis is that higher values of this are associated with higher values of capital rewards, "r".

j. Average product of labor (APL). As a measure of productivity, this variable is part of neoclassical theory, although not as controversial as marginal productivity. In fact, it is probably the most commonly used measure of productivity in the popular press and in government reports because of its simplicity. For this paper, it is defined as GDP per labor hour, and it is hypothesized to have a positive relationship with labor rewards, *ceteris paribus.*

k. Average product of capital (APK). Like APL, this variable is a concept within neoclassical economics, and it is hypothesized that higher APK values will be associated with higher capital returns, all else constant. It is operationalized as GDP per value of total of fixed assets percent.

l. Variable Returns to Scale Efficiency Score for Labor (VRS Efficiency of Labor). This is used to predict labor income share per nation, although it assumes VRS among the nations in producing output, which is not part of the neoclassical assumption of marginal productivity, but it is used here as point of comparison with CRS efficiency. The greater the VRS efficiency of labor is, the higher "w" is, all else held constant.[8]

m. Constant Returns to Scale Efficiency Score of Labor (CRS Efficiency of Labor). This is used to predict labor income shares per nation, and it is consistent with the neoclassical assumptions of constant returns to scale when it comes to factor productivity. The higher CRS efficiency is, the higher labor income share should be, everything else held constant.

[8] VRS efficiency allows for increasing and decreasing returns to scale to be used by a DMU to achieve efficient output. CRS efficiency only allows for constant returns to scale in production to be used by a DMU to attain efficiency, which is a more restrictive assumption with regard to efficiency.

n. Variable Returns to Scale Efficiency Score of Capital (VRS Efficiency of Capital). This variable is used to predict NOS/Fixed Assets percentage per nation, although it does not adhere to neoclassical assumptions of constant returns to scale in production when it comes to estimating factor payments. The greater VRS efficiency of capital is, the higher NOS/Fixed Assets percentage is on average, *ceteris paribus*.

o. Constant Returns to Scale Efficiency Score of Capital (CRS Efficiency of Capital). This is used to predict NOS/GDP percentage per nation and adheres to standard neoclassical assumptions of constant returns to scale in production. The greater the value of this variable, the higher NOS/Fixed Assets percentage should be on average, all else constant.

Results

(Insert Table 2 around here)

Table 2 shows the regression model results for two sets of models—one set (Models 1 to 4) with labor income shares for each nation as the dependent variable, and the other set (Models 5 to 8) with NOS/Capital Percentage as the dependent variable. Model 1 shows that the simple Marginal Product of Labor variable is not statistically significant, and so a neoclassical economics concept is not supported. Likewise, the overall Heritage Index score does not work well as a predictor either, and does not work well in Models 2, 3 and 4 as well, just as the unemployment rate does not work in Models 1 to 4. All other variables in Model 1 are statistically significant, and some have their predicted signs, although average annual labor hours worked, trade union density, and college graduates have signs opposite of what was hypothesized. This is the case not only for Model 1 but

also for Models 2 to 4, although the college grads variable was not statistically significant in Model 2. There were no signs of multicollinearity among the independent variables after examining a Pearson correlation matrix and variance inflation factors (Levine, Stephan, Krehbiell, and Berenson 2008 pages 625-626, Anderson, Sweeney and Williams 2010), so other explanations must be sought for why the hypotheses regarding the variables' signs are not supported.

With regard to average annual hours worked, the Marxian concept of longer labor hours is not shown to result in less labor income in the aggregate, although there is an inverse relationship between longer work hours per worker per year and higher average annual wages per worker. As mentioned earlier, average annual pay nor estimated average hourly pay were used as dependent variables because of problems with autocorrelation in the models. Average annual work hours were the highest in Korea, Hungary, and the Czech Republic (the top three), yet these nations had high labor income shares, probably because unlike some of the other nations used in the analysis, these countries have shorter vacation times and less paid holidays with low average productivity rates (Stephenson 2012). They have a greater percentage of the adult population employed on average because of less generous social insurance programs, and longer work weeks on average when compared to nations such as Germany, France and Belgium who came in toward the low end of average annual hours worked (Birchfield and Crepaz 2002, Lambert 2011). Therefore, although average pay may be slightly lower, labor income share is fairly high because of the greater hours and greater participation rates, which basically amounts to some type of exploitation when one compares different nations.

The negative signs for trade union density and college graduates in the first three models could also be explained along the same line of reasoning for average annual hours worked. The nations with the greatest labor income shares also have the lowest trade union densities. Unions achieve higher wages and fewer working hours and better benefits for their workers so that long work hours are not

needed to accomplish a certain standard of living for most workers. The smaller the union presence in a nation, the less impact unions will have, which means workers work longer hours and receive smaller paid leave and paid vacation benefits. Likewise, those nations with high average annual pay and salaries also have higher percentages of college grads in their labor forces. Yet, these nations have lower labor income shares compared to those nations with less college grads because of shorter work weeks, earlier retirement of workers, etc.

The other statistically significant variables in Models 1 to 4 have their hypothesized signs with the Marxian profit to wage variable having the largest impact of all variables in each model as indicated by the high beta (standardized regression coefficient) scores. The second highest beta for Models 1, 3 and 4 is for average annual hours worked, another exploitation concept. The inflation rate, as expected, has a negative effect on labor income shares. Model 2 shows that the average productivity of labor (APL) variable does not work well in predicting labor returns—it has a negative coefficient and is not statistically significant. In Models 3 and 4, the VRS and CRS efficiency scores for labor show mixed results. Only the VRS efficiency score is statistically significant, and so some type of productivity measurement is supported, although not the traditional marginal productivity concept. Higher variable returns to scale efficiency scores are associated with higher levels of labor income shares on average and, holding all else constant.

Recall from the discussion of DEA that each DMU or nation is ranked according to how well its inputs (total number of workers and total labor hours) are minimized in achieving maximum output (GDP) when comparing each nation to others. A higher score means less workers and total hours used to achieve GDP relative to other nations, which implies or means fewer labor hours worked per worker to achieve higher GDP. The CRS efficiency score is not linked to labor income shares, and this is not

surprising given that on average, greater labor hours per worker are associated with higher labor income shares.

The VRS scale score works better as a predictor perhaps because variable returns to scale allows for increasing and decreasing returns to scale in production, which makes it easier for a DMU to demonstrate efficiency, and this means that these efficiency scores are higher on average than those for CRS efficiency (see Appendix A for an example that uses 2010 DEA results). Therefore, greater labor hours and a greater number of workers used by one nation when compared to some other nations would be deemed efficient under variable returns to scale if a large enough GDP was produced, assuming decreasing returns to scale in production would be permitted, which it is under the variable returns to scale DEA model. That is, a nation like the US would be deemed efficient most years under VRS DEA because, although it uses the greatest amount of inputs among the nations, it also produces the greatest amount of output, although its production would be characterized by decreasing returns to scale—greater inputs yield lesser output relative to other nations given the size of the inputs.[9]

Of course, variable returns to scale violate neoclassical assumptions of marginal productivity (Keen 2011, pages 94-95). Only constant returns to scale can be used in marginal productivity analysis so as to account for all factor payments and to uphold to the concept or principle of competitive markets. However, CRS efficiency does not show any predictive ability in Model 4. Therefore, most neoclassical assumptions do not hold given the results of Models 1 to 4.

[9] Recall that the definition of efficiency used in this paper is in the tradition of the narrow, neoclassical concept of productive efficiency. There was no consideration of negative externalities such as pollution as a production outcome, although negative externalities could be explained perhaps by the decreasing returns to scale score received by the US and others in that some waste or pollution occurs in the generation of output.

All three models used in predicting labor income shares explain over 50% of the variation in labor income shares. The Marxian/heterodox concepts of exploitation seem to do better in explaining labor income shares than the other measures mentioned.

In looking at Models 4 to 8 where the dependent variable is a measure of returns to capital or fixed assets, the overall Heritage Index score (an indicator of pro-business, laissez faire policies in a nation) is statistically significant in all four models and has its expected sign—greater laissez faire results in greater returns to capital. The inflation rate variable has a negative sign as expected as one would expect lower returns to capital the greater the inflation rate, all else held constant. College grads as a percentage of the labor force are associated with higher returns to capital on average, although this only works in Model 5 and is not statistically significant in Models 6 to 7. If college grads are associated with lower labor income shares, then it makes sense that they would be linked to higher capital returns—less aggregate factor payments going to labor means more going to capital, all else held constant. It could also imply that college grads yield high productivity rates for corporate employers. Of course, not too much can be made of this variable since it only predicts well in one of four models.

The unemployment rate variable is statistically significant in Models 5, 6 and 8 where a higher unemployment rate is associated with lower returns on average. Corporate profits/returns are impacted by unemployment rates, implying a lower amount of consumer demand. Meanwhile, the trade union density variable does not work at all in any of these models except in Model 6. Capital returns are not impacted by a greater or lesser degree of unionization in any of the nations. On the other hand, the exploitation variables work well—on average, the greater the profit to wage ratio and the greater the average annual hours worked, the greater the return on capital in each nation. Getting more work hours from the labor force or paying less annually per hour or per year results in greater

surpluses for business enterprises as wages are "squeezed" or as labor has to work more hours. The profit to wage variable has the highest standardized regression coefficients in each model and therefore can be considered the strongest predictor among the list of variables.[10]

Finally, the simple MPK variable in Model 5 does not work well, somewhat weakening neoclassical arguments for a marginal revenue product of capital concept. However, the VRS and CRS efficiency scores work well and have the second highest betas in their respective models, 7 and 8. The CRS scores support neoclassical notions of requiring constant returns to scales to explain factor payments, although with the VRS scores increasing and decreasing returns to scale explain returns to capital quite adequately as well. The less capital inputs used given a certain level of GDP yielded, the greater the efficiency score, and the greater the return on capital on average. In looking at the example provided in Appendix A for 2010, the two sets of scores for capital efficiency were actually not that far apart in their values when compared to the CRS and VRS scores for labor efficiency. It appears that "r" is maximized whether one assumes increasing, decreasing, or constant returns to scale, which implies that constant returns to scale alone are not needed to yield efficiency or appropriate rewards to capital. The models which employ VRS and CRS capital efficiency scores also yield a high degree of explanation in the variance of capital returns with adjusted r-squared values greater than 70%. However, Model 6, which uses GDP as a percentage of total fixed assets (APK) yields an even higher explanation of variance (around 92%), and APK has the second highest standardized regression coefficient among the variables.

[10] Of course, neoclassical economists would probably point out that high returns on capital drive the ratio of NOS to wages, and not necessarily the other way around. Shrewd capital investment on the part of businesses and investors would lead to higher rewards on capital, and so investors should receive higher income shares than labor than would otherwise be the case. In doing two stage least squares regression to control for possible simultaneity bias, this was not found to be true, although the model suffered from serial correlation.

In Models 5 to 8, neoclassical models get more support than what they received with regard to predicting labor income shares, although the results are mixed. VRS efficiency is a good predictor of capital returns, but it cannot be permitted under neoclassical assumptions, and average product of capital is not a central concept in the controversy over capital. The focus of the controversy has always been on marginal productivity where constant returns to scale is a core assumption/requirement of the theory. The simple marginal product of capital variable does not work, although the CRS variable does, so the models show mixed results for neoclassical theory.

Conclusion

The results of this paper fail to show support for a strong theory of macroeconomic marginal productivity when it comes to income shares or rewards to labor and capital. The key variables of marginal product, average product, and CRS efficiency are not effective in predicting labor income shares. Only VRS efficiency does well in predicting labor income shares, which flies in the face of neoclassical assumptions and theoretical requirements of competitive markets and constant returns to scale. The simple marginal product of capital variable employed to predict capital returns also does not work well, although the variables APK, CRS efficiency, and VRS efficiency work well. Again, however, only CRS efficiency embodies a key neoclassical assumption whereas the other two do not. Most importantly, it is only with APK that a substantive amount of the variation in capital returns is explained by a productivity variable alone (33%), although it comes in second after the profit to wage ratio variable when step-wise regression is employed. In bivariate regression models, the VRS and CRS efficiency variables that were statistically significant in the multivariate Models 1 to 8 predicted lesser amounts of variation in labor and income shares. Finally, it should be remembered that VRS and CRS efficiency scores are *relative* measures of efficiency brought about by comparisons made among different DMUs. Usually marginal product is discussed or taught with reference to looking at

the productivity of a single nation over time or by tracking the capital accumulation of different nations over time but never saying which nation has higher marginal productivity. In the list of the 15 and 16 nations used in this analysis, if more nations were added as data became available, the efficiency scores would have to change if only slightly.

There does seem to be more consistent findings in the models that support Marxian and heterodox approaches to explaining labor and capital. The profit to wages variable was statistically significant in each model and consistently had the highest standardized regression coefficients. This would seem to support the notion that payments to the factors of production are more matters of allocation and distribution according to class rather than productivity. This is underscored with regard to capital rewards in that the Heritage Index overall score variable worked well in all four capital returns models. This score gives nations higher rankings according to the strength of their property rights, investment freedom, and the presence of limited government. Such institutions favor capital, and so one could expect that capital rewards would be higher in nations with higher rankings on these dimensions regardless of capital's productivity, all else held constant.

Of course, the heterodox concepts do not individually explain all of the variation in factor payments either. Perhaps a compromise position would be that neoclassical assumptions are only valid if marginal productivity is loosely defined as something close to or the same as average productivity and that class and property relations have to be acknowledged in income distribution. Some introductory textbooks mention that pinpointing concepts such as marginal cost, marginal revenue, and marginal product are elusive in reality and that only average measurements are used in practice.

If so many critics consider marginal productivity to be an unrealistic concept, and if the only results that seem to support it are based on data that form an accounting identity (Q = wL + rK), why is it still taught in many universities and still used in some macroeconomic textbooks? Perhaps one possible scenario in the future is that economic or political economy pedagogy and discussion take on more diverse views and admit that property and class relations have to be discussed along with productivity concepts, although average productivity should be emphasized and not marginal productivity.

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Table 1

Nations used in DEA and Regression Analysis

For Labor:	**For Capital:**
Australia	Australia
Austria	Austria
Belgium	Belgium
Canada	Canada
Czech Republic	Czech Republic
Finland	Finland
France	France
Germany	Germany
Hungary	Hungary
Korea	Korea

Luxembourg	Luxembourg
Netherlands	Netherlands
Switzerland	Slovenia
United Kingdom	Switzerland
United States	United Kingdom
	United States

Table 2

Dependent Variable: Labor Income Share

	Model 1			Model 2			Model 3			Model 4		
	b	s.e.	Beta	b	s.e.	Beta	b	s.e.	Beta	b	s.e.	Beta
(Constant)	64.795	4.789		68.004	6.851		48.52	7.222		56.319	6.847	
Overall score	0.051	0.059	0.052	0.044	0.060	0.046.	0.049	0.059	0.052	0.075	0.061	0.079
Inflation Rate CPI	-1.223***	0.237	-0.308	-1.244***	0.241	-0.314	-1.149***'	0.244	-0.288	-1.186***	0.249	-0.297
College grads	-0.073**	0.041	-0.104	-0.064	0.045	-0.092	-0.105**	0.04	-0.157	-0.094**	0.041	-0.14
Unemp. Rate	-0.337	0.176	-0.11	-0.341	0.182	-0.111	-0.217	0.168	-0.075	-0.231	0.176	-0.08

Trade Union Density	-0.055**	0.022	-0.141	-0.056*	0.022	-0.144	-0.053**	0.021	-0.143	-0.062***	0.021	-0.166
Profit to Wage Ratio	-0.528***	0.043	-0.685	-0526***	0.044	-0.683	-0.49***	0.043	-0.643	-0.513***	0.042	-0.672
Average annual hours worked	0.01***	0.002	0.345	0.009***	0.002	0.306	0.014***	0.002	0.546	0.011***	0.002	0.43
MPL Chg. Q/Chg. Pay	0.008	0.007	0.062									
APL (GDP/labor hour)				-0.025	0.045	-0.050						
VRS Efficiency of Labor							9.039***	3.082	0.259			
CRS Efficiency of Labor										5.329	3.096	0.133
	Adj. r-square: 0.53			**Adj. r-square: 0.53**			**Adj. r-square: 0.56**			**Adj. r-square: 0.54**		
	D-W stat: 2.18			**D-W stat: 2.17**			**D-W stat: 2.25**			**D-W stat: 2.34**		
	N = 165			N = 165			N = 165			N = 165		

**p < 0.05

***p < 0.01

Dependent Variable: NOS / Fixed Assets %

	Model 5			**Model 6**			**Model 7**			**Model 8**		
	b	**s.e.**	**Beta**	**b**	**s.e.**	**Beta**	**b**	**s.e.**	**Beta**	**B**	**s.e.**	**Beta**
(Constant)	-2.561	1.044		-5.213	0.491		-9.767	1.23		-9.877	1.149	
Overall score	0.03**	0.013	0.123	0.015**	0.006	.062	0.036***	0.011	0.148	0.036***	0.011	0.15
Inflation Rate CPI	-0.247***	0.053	-0.25	-0.088***	0.024	-.089	-0.144***	0.045	-0.145	-0.117***	0.044	-0.119
College grads	0.029***	0.009	0.168	.005	0.004	.026	-0.005	0.009	-0.026	-0.01	0.008	-0.057
Unemp. Rate	-0.091**	0.038	-0.119	-0.074***	0.017	-.097	-0.062	0.032	-0.081	-0.078**	0.031	-0.102
Trade Union Density	-0.005	0.005	-0.053	-0.011***	0.002	-.109	-0.006	0.004	-0.058	-0.002	0.004	-0.021
Profit to Wage Ratio	0.115***	0.01	0.599	0.127***	0.004	.662	0.14***	0.008	0.729	0.139***	0.008	0.723
Average annual hours worked	0.002***	0	0.231	0.001***	0.001	.165	0.002***	0	0.255	0.002***	0	0.253

MPK Chg. Q/Chg. Fixed Assets	0.105	0.073	0.069								
APK (GDP over Total Fixed Assets pct.)				0.143***	0.006	0.587					
VRS Efficiency of Capital							7.104***	0.845	0.414		
CRS Efficiency of Capital										7.431***	0.788	0.455
	Adj. r-square: 0.64			**Adj. r-square: 0.92**			**Adj. r-square: 0.75**			**Adj. r-square: 0.76**		
	D-W stat: 2.64			**D-W stat: 1.85**			**D-W stat: 2.46**			**D-W stat: 2.40**		
	N = 168			N = 168			N = 168			N = 168		

**p < 0.05

***p < 0.01

No signs of multicollinearity given Pearson r's and VIFs.

Appendix A

2010 DEA Scores for OECD nations

Capital Efficiency Scores

Inputs	**Outputs**
Consumption of Capital Thousands $	GDP 2010 Thousands $
Gross cap formation Thousands $	
Fixed Assets Thousands $	

	Input-Oriented	***Input-Oriented***

DMU No.	*DMU Name*	*VRS Efficiency*	*CRS Efficiency*
1	Australia	0.86727	0.86543
2	Austria	0.78252	0.77607
3	Belgium	0.90248	0.89726
4	Canada	1.00000	1.00000
5	Czech Republic	0.68465	0.67054
6	Finland	0.96588	0.94471
7	France	0.89486	0.89394
8	Germany	0.90382	0.89970
9	Hungary	0.86399	0.82246
10	Korea	0.99487	0.99319
11	Luxembourg	1.00000	0.94046
12	Netherlands	0.87702	0.87175
13	Slovenia	0.88662	0.79520
14	Switzerland	1.00000	1.00000
15	United Kingdom	1.00000	1.00000
16	United States	1.00000	1.00000

Labor Efficiency Scores

Inputs	**Outputs**
Tot Employment Thousands	GDP 2010 Thousands $
Total Hours Worked Thousands	

DMU No.	*DMU Name*	*Input-Oriented VRS Efficiency*	*Input-Oriented CRS Efficiency*
1	Australia	0.81801	0.66584
2	Austria	0.85275	0.69003

3	Belgium	0.98328	0.79453
4	Canada	0.78337	0.64058
5	Czech Republic	0.51611	0.44709
6	Finland	0.76818	0.64937
7	France	0.98179	0.77249
8	Germany	0.97073	0.76061
9	Hungary	0.51191	0.44690
10	Korea	0.57889	0.49097
11	Luxembourg	1.00000	1.00000
12	Netherlands	1.00000	0.81309
13	Switzerland	0.84370	0.67875
14	United Kingdom	0.79683	0.63296
15	United States	1.00000	0.84446