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A simple model of bubble formation

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The study of bubbles has become increasingly important in recent years, not least because of the perception that the financial crash of 2007-2009 involved one or more of them. Several bubbles have occurred in recent decades, and some observers believe that their frequency and/or severity may be increasing.

One definition of a bubble is a "trade in high volume at prices that are considerably at variance from intrinsic value" [1]. This is symmetric: "at variance" could mean that prices are either too high or too low. On the other hand, asymmetric definitions include those of Kindleberger [2] and Shiller [3]: respectively "a sharp rise in price of an asset or a range of assets in a continuous process, with the initial rise generating expectations of further rises and attracting new buyers", and "a situation in which temporarily high prices are sustained largely by investors' enthusiasm rather than by consistent estimation of real value" (the latter definition is of a *speculative* bubble, but speculation could in principle equally apply to downward as to upward deviations). The asymmetry accords with the metaphor of a "bubble", suggesting something that grows steadily and then bursts suddenly. It is an empirical question which is more accurate; is it true that in practice important systematic movements in price are always, or generally, upward? Or are they equally in either direction? Possibly the downward movement occurs but is ignored because it is thought to be less important in practice? In addition, it is possible that the phenomena of bubbles and fluctuations represent distinct mechanisms, the latter being not only symmetric, but also less extreme and perhaps more regular [4].

This question of symmetry or asymmetry is highly relevant to the way bubbles are explained, modelled and empirically tested. The classic models in the literature mainly tend to take a symmetric view, focusing on price fluctuations, volatility, overreaction and autocorrelation, which do not distinguish upward from downward movements. Caginalp and Ermentrout [5] propose a psychological model of fluctuations in financial markets, driven by memory of price history with exponential decay, and the possibility of capitalising on the difference between price and intrinsic value. They test this by examining overshoot in the return to equilibrium after an exogenous deviation from intrinsic value [6]. Shleifer and Summers provide an account based on noise traders which is entirely symmetric, although four of their nine historical examples are of price levels above fundamentals, the other five being symmetric fluctuations, and none being downward movements [7]. Similarly, the intrinsic bubble model of Froot and Obstfeld [8] and the contagion model of Topol [9] are both symmetric, as are the psychological theories of adaptive expectations or investor confidence [see ref. 3, pp 69-71]. In contrast, Diba and Grossman argue that bubbles in real stock prices can never be downward, because their rational expectations model would generate negative prices, which are impossible [10].

This paper presents a simple model that predicts that bubbles only occur in markets with rising prices, which may correspond better to the historical record than the symmetric approach. It requires modification of the idea of rationality, but not its radical abandonment.

Free-floating and cost-tethered markets

Another implication of the definitions given above is that the central property of a bubble concerns how prices are set: how do the seller and the potential buyer come to form a compatible perception of "the going price"? This paper presents a model of price-setting in the context of a market where the price is not set in direct relation to costs. In such a situation, information is required on what is the "going price", which depends only on the willingness of the potential participants in a market exchange, based on whatever preferences and information they hold. This applies to financial and property markets, as well as to some new products, and to collectibles. The bubbles literature is focused on financial and property markets [1-14], but without analysis of why these and not markets for established goods and services; in addition, new products may have played an important part in the dot-com boom of the 1990s, as well as in the 1920s, because it is difficult to judge their fundamentals [11].

Trend extrapolation

Whether or not a bubble arises depends on ***what information is used*** to set "the going price" that buyer and seller can agree on. One possibility is that it is influenced by the extrapolation of a trend in people's minds; in principle it could be a rising or a falling trend. There is abundant evidence that this can occur [3,7,11,12]. Shleifer and Summers stated: "One of the strongest investor tendencies documented in both experimental and survey evidence is the tendency to extrapolate or chase the trend" [7].

With a perceived trend, the going price is related to the timing of the transaction. For example with a rising trend a potential buyer will wish to buy sooner, and a potential seller will be tempted to wait for a higher price; and *vice versa* for a downward trend. This creates a tendency towards a sellers' market in the former case, with the buyer needing either to pay a premium so as to buy now, or to wait – in which case the price will have risen in any case; the seller is in the position of being able to sell at this higher price, or wait until prices rise. Demand is increased and supply is restricted. The overall effect is that the perception that prices are set to rise brings about a higher price "now", i.e. this is a self-fulfilling prophecy: a perceived rising price trend fuels a further rise in the price. The argument is reversed with a price that is perceived as being destined to fall. This explains why eagerness to buy is observed in rising markets, e.g. in property, and eagerness to sell is a feature of falling ones, without necessarily involving "speculation". In either case the emotion (eagerness) is secondary to the interpretation (a trend that will continue) of the available information (a past trend; possibly with additional "stories", e.g. that property is always a good investment).

The model

A simplified way of representing a bubbles-prone market is to start from a standard model of market equilibrium, assuming for simplicity that the demand and the supply curves are both linear and given by the parameters a,b,c,d with b,d > 0:

$$D(P) = a - bP \tag{1}$$

$$S(P) = c + dP \tag{2}$$

where P is the price, and D(P) and S(P) respectively represent the willingness to buy and to sell the asset now at the existing price.

If it is perceived that a price trend is expected to continue into the future (trend extrapolation condition), the price increment ΔP brought about "now" by this perception can be represented by:

$$\Delta P = \theta P, \tag{3}$$

where θ is the proportional expected future price increment given by

$$\theta = f(P') \tag{4}$$

with f(.) an increasing function and P' a time derivative. Therefore, $\theta<0$ for $P'<0$, and $\theta>0$ for $P'>0$. For simplicity the perceived trend P' is regarded as linear. The price "now" modified by trend extrapolation is $(P + \Delta P)$. For a representation of how θ may relate to P' in terms of ζ, the tendency to buy or "investor sentiment", see Caginalp and Ermentrout [5]; Shiller [12] has something similar. Up till now, the model and its two preceding conditions are entirely symmetric.

The original equations now become:

$$D(P) = a - bP + m\theta P \qquad (1')$$

$$S(P) = c + dP - n\theta P \qquad (2')$$

where m,n are parameters (>0) that determine the extent to which the price is affected by trend extrapolation. (Traditionally, equations (1) and (2) are derived from optimization; similarly, it would be possible to derive θ in terms of optimization in the presence of a financial constraint.)

This gives two conditions:

if $m\theta > b$, a rise (fall) in P will lead to a rise (fall) in D(P), and (5)

if $n\theta > d$, a rise (fall) in P will lead to a fall (rise) in S(P). (6)

Under these conditions, therefore, instead of the usual decreasing function in P for D(P) and increasing function for S(P), as represented by equations (1) and (2), the situation is reversed. As each of the new terms increases in absolute magnitude away from equations (1) and (2), the supply and demand curves that the equations represent rotate towards or beyond the vertical (with price on the y-axis, as is usual in economics).

Equations (5) and (6) can be written respectively as $\theta>b/m$ and $\theta>d/n$. Because $b/m>0$ and $d/n>0$, this model predicts asymmetry in the sense that the property of reversing the overall direction of equations (1) and (2) only occurs when $\theta>0$ and thus also $P'>0$. In the case where $\theta<0$, the terms $m\theta P$ and $n\theta P$ would have the same signs as bP and dP respectively, so that they would merely accentuate the normal decreasing and increasing functions represented respectively by equations (1) and (2). A bubble can therefore only occur with a (perceived) rising market, and represents the sellers' market as described above: the model predicts that if prices start at or near their intrinsic level, a self-fulfilling and thus self-perpetuating tendency will tend to occur in markets with an upward – but not a downward – price trend.

The model shows clearly what would be predicted to happen either if $\theta>b/m$ ***and*** $\theta>d/n$ or if $\theta<b/m$ ***and*** $\theta<d/n$. What if one of these conditions is met but the other is not? Both are driven by θ, and therefore expected to move in the same direction, but the point at which $\theta=b/m$ and $\theta=d/n$ are not necessarily reached at the same moment. Interpreting equations (1') and (2') respectively as demand and supply curves, a bubble can be said to start when the gradients of these curves pass the point where their gradients are equal. Without any trend extrapolation, clearly $d>-b$; and with mild trend extrapolation, it is still true that $d-n\theta>m\theta-b$. This gives a condition for a bubble when $d-n\theta<m\theta-b$, or $d+b<\theta(m+n)$, i.e. when

$$\theta > \frac{b + d}{m + n} \qquad (7)$$

The conditions specified by the model bring about a *bubble equilibrium*, in which a rising price reinforces the perception that prices are destined to rise, and in turn this perpetuates the rising trend.

Comments on the model

The metaphor of a bubble suggests that its bursting is sudden, but this may not be empirically accurate. Such bursting is not directly covered by the model; the main prediction is that the price will deviate ever more from the intrinsic value with the passage of time, and that a point is reached where this cannot continue. Bubble equilibria are therefore inherently unstable. The return towards the intrinsic value then proceeds according to the standard equations (1) and (2), possibly accentuated by downward trend extrapolation, and is not necessarily sudden.

In relation to what could precipitate a bubble ending, one factor is simply the distance of the perceived going price from fundamentals; in situations where there is no clear intrinsic value, as in the case of real estate, the equivalent role would be played by the realisation that prices have risen to a point that is no longer affordable so that the risk of default is high. But in addition, because the upward movement in the model is driven by perception of future profitability, it is likely that information directly contradicting upward trend extrapolation would be especially potent, as appears to have been the case in October 1929 [11].

The value of θ has been treated as uniform across the whole population, but this is not necessarily the case. An alternative conception is possible, but not developed in this paper, of variable individual responses to external "pressure". This pressure is seen as uniform, manifest as a societal perception that the past trend will continue, in the form of stories in the media, etc. An alternative conception is that of Keynes: people base their decisions on "what average opinion expects average opinion to be" [15]. The varying responses could be represented by a distribution; there is evidence from experimental economics studies that such variation does occur [13]

In summary, this simple model predicts that under conditions of trend extrapolation, where the going price is affected by perceptions of a future price rise – but not of a fall – the result will be an ever-increasing price if the conditions specified in equation (7) are met. Eventually the deviation of the price from the intrinsic value leads to a point at which reversal occurs, a return towards that intrinsic value. If the assumption of linear functions is relaxed, the functional form could affect when/how this point is reached, quantitatively modifying the rapidity and timing of events but not affecting its basic properties.

This simple model could be applied to e.g. financial or property markets, but it ignores real-world complexity, not least that they tend to be inter-related, because a rising property market tends to involve a financial bubble in willingness to lend, along with that occurring in property prices themselves. It is likely to be the unsustainability of this secondary financial bubble that causes a property bubble to burst.

Situational rationality

The key element driving this model is trend extrapolation, which some authors have described in terms of a radical departure from rationality, e.g. "emotional" (Caginalp & Ermentrout) [5]. There is little doubt that markets such as housing are substantially influenced by exogenous psychological factors [12,14]. But a strong role for irrationality is hard to reconcile with the experimental finding that individuals behave more in accord with individual-rationality models when in the social context of exchange institutions than when studied alone [16]. Some authors whose work is closely connected with the actual functioning of markets tend to use terms such as "group-think", "optimism" or "panic" [3,14]. Often these descriptions refer to situations with a relatively small group of operators such as financial market traders, who have substantial face-to-face contact. Under such conditions it is quite likely that emotions do become involved – but whether they play a ***causal*** role is unclear.

In particular, some markets can involve trend extrapolation but without these features. An example is of someone, not a professional investor, buying a property to live in rather than primarily for investment. If the consensus is that property prices are destined to rise for at least the foreseeable future, this forms an important element in the ***calculation*** (which does not involve "speculation") of which property should be selected, and with what degree of urgency. In such a situation rising prices would not be a source of excitement, rather they might pose a threat, but acting as a constant reminder not to delay rather than playing a motivating role in price-setting behaviour. Are professional market traders "less rational" than this home buyer? And if emotion is not a necessary cause in the housing market, why introduce it as such into financial markets?

The view of decision making proposed here involves a rational calculation, but one that takes into account a wider range of information than in the standard view of rationality. The use of this extra information results from the recognition that perfect information and especially perfect foresight are not features of real markets, so that this could be seen as a form of bounded rationality – although limitation of human calculation ability is not a central feature here. Where there is a shared public perception of a trend, this is external to each market participant, so that they all are compelled to participate in the bubble-fuelled behaviour even if privately they perceive that they are in a bubble that is likely to burst soon. This interpretation accords with the observation that under such circumstances, failure to participate is met with sanctions that negatively affect career prospects.

The limitation is not on rational calculation ability, but rather on the use made of the type of information that is available; this accords with the observation that market participants may prefer to employ simple reasoning strategies based on this type of limited information even when calculation of the fundamental values is possible, as appears to occur regularly in experimental studies. As Brav and Heaton say, it is difficult to distinguish between “behavioral theories built on investor irrationality and rational structural uncertainty theories built on incomplete information about the structure of the economic environment” [17].

The term “situational rationality” has been used by previous authors such as Schutz, to indicate that decision making has to take account of common knowledge. However, it has tended to take on a sense that is further removed from rationality in the sense of deliberate calculation than is intended here. For example, it has been used to challenge the assumptions that behaviour is foresightful and that rationality occurs in advance of action, whereas the model proposed in this paper requires a forward-oriented calculation but with inability to prophesy the future (for a useful review see Townley [18], who deals with institutional, contextual and situational rationality as three forms of embedded rationality).

Conclusion

The argument in this paper is that the following conditions are necessary for bubbles to form:

1. the information that sellers and buyers use to agree “the going price” is based on trend extrapolation
2. quantitatively, a bubble will form if

$$\theta > \frac{b + d}{m + n}$$

3. *a fortiori*, P’ > 0.

Each on its own is not sufficient, but if all are present then this is a sufficient condition for bubble formation. Clearly this is a simplification, because real-life economic systems are subject to additional exogenous influences.

The model predicts asymmetry: the process of progressive deviation from intrinsic value (or from affordability) will occur in a rising but not a falling market, and actual prices will be higher than intrinsic value in such circumstances.

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