Labour values, prices of production and market prices in the German economy

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Summary

According to the work of Shaikh (1984), Cockshott et al. (1995) and Cockshott & Cottrell (1997, 2003) simple labour theory of value and neoricardian theory are tested with GLS regression models using German Input-Output tables. Farjoun's and Machover's (1983) suggestions on Probabilistic Political Economy are applied as well. Both approaches yield very good results in explaining data. Differences in estimation outcomes are mainly negligible. But there is one crucial point: Although a certain transformation tendency exists, profit rates and capital intensity seems to be negatively correlated. Hence, simple labour theory of value is superior in explaining reality. Moreover, the German economy seems to be in a state of statistical equilibrium during the years 2000 and 2004.

1 Introduction

In non-mainstream economic theory there are usually two ways of explaining market prices: First, labour theory of value which states that prices are driven by vertically integrated labour time (labour values). This approach, originally used by Karl Marx in "Capital, Volume 1", evoked the famous transformation problem because an equilibrium profit rate seems to exist only in case of uniform capital intensity or zero profits. Second, the discussion about labour values has let to the development of neoricardian prices of production based on the work of Pierro Sraffa and his followers. These authors believe the transformation debate reached its well deserved end because their model provides prices generating an

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equilibrium profit rate. Hence, it is typically viewed as state of the art and even prominent marxian authors stated that labour values "play no role whatsoever in the discussion of exchange and price" (Roemer 1981, p. 200).

On the other hand, there is a growing body of empirical studies claiming that deviations from values to prices are quite small (see Shaikh (1984), Petrović (1987), Ochoa (1989), Cockshott et al. (1995), Cockshott & Cottrell (1998), Tsoulfidis & Maniatis (2002), Cockshott & Cottrell (2003)). The authors found correlation coefficients and coefficients of determination R^2 to be considerably larger than 0.9. Therefore, labour values might be as good in explaining market prices as neoricardian prices of production are. Furthermore, empirical results could be substantiated by theoretical arguments developed by Farjoun & Machover (1983) and Shaikh (1984). In view of the traditional approaches, this is a serious challenge. Unsurprisingly, methodological critique has taken place to doubt these outcomes (Steedman & Tomkins 1998; Kliman 2002, 2005; Díaz & Osuna 2005–06, 2007). We will deal with this in more detail later on.

The aim in this paper now is as follows: Based on a brief sketch of theoretical basics in the following section, especially those of Farjoun & Machover (1983) and Shaikh (1984), we should investigate the empirical relationship between labour values, prices of production and market prices in the German economy. For this reason, in the third section we will carry out regression analysis on several empirical models to check how well theories fit the data. Density functions for relevant variables are estimated and compared to theoretical prediction, too. Afterwards, as usual, a conclusion will be given.

2 Theoretical framework

2.1 The law of value

Consider an economy with n sectors and a uniform period of production.[1] Each sector is producing a single output. The economy is described by a linear, constant-returns-to-scale technology $\{\boldsymbol{A}, \boldsymbol{l}\}$, where $\boldsymbol{A} = (a_{ij})$ is an indecomposable, productive $(n \times n)$-Matrix of input coefficients and $\boldsymbol{l}$ is the $(1 \times n)$-vector of direct labour inputs.[2] Labour value $\lambda_i, i = 1, \ldots, n$, is the sum of direct and indirect labour inputs needed to produce commodity i with respect to $\{\boldsymbol{A}, \boldsymbol{l}\}$. Therefore, the $(1 \times n)$-vector of labour values $\boldsymbol{\lambda}$ is obtained by the following equation:

[1]For the usual framework of marxian economics see for instance Pasinetti (1977), Roemer (1981) or Mohun (2004).

[2]Every matrix, vector and scalar used in this paper is real and nonnegative.

$$\boldsymbol{\lambda} = \boldsymbol{\lambda A} + \boldsymbol{l}. \tag{2.1.1}$$

Since we have assumed $\boldsymbol{A}$ to be indecomposable and productive we may rewrite (2.1.1) as

$$\boldsymbol{\lambda} = \boldsymbol{l} \left(\boldsymbol{I} - \boldsymbol{A}\right)^{-1}. \tag{2.1.2}$$

For a moment we are adopting that the whole net product of the economy is paid to workers because there are no capitalists. In this case prices are determined by labour values:

$$\boldsymbol{p}^e = \boldsymbol{p}^e \boldsymbol{A} + w^* \boldsymbol{l}, \tag{2.1.3}$$

$$w^* = \boldsymbol{p}^e \boldsymbol{y} = 1, \tag{2.1.4}$$

where $\boldsymbol{p}^e$ denotes the $(1 \times n)$-vector of "exploitation" prices and $\boldsymbol{y}$ is the $(n \times 1)$-vector of net product. Applying (2.1.4) to (2.1.3) and recalling (2.1.1) immediately shows that

$$\boldsymbol{p}^e = \boldsymbol{\lambda}. \tag{2.1.5}$$

But in reality, a certain fraction of net product goes to capitalists simply because they are commanding the means of production. Workers receive a subsistence wage basket instead of w^*, i.e. a $(n \times 1)$-vector of commodities $\boldsymbol{b}$. Therefore,

$$w = \boldsymbol{p}^e \boldsymbol{b} = \gamma w^*, \ \gamma < 1, \tag{2.1.6}$$

$$\frac{1}{\gamma} w = w^*. \tag{2.1.7}$$

By rearranging (2.1.3) we obtain

$$\boldsymbol{p}^e = \boldsymbol{p}^e \boldsymbol{A} + \frac{1}{\gamma} w \boldsymbol{l}, \tag{2.1.8}$$

$$\Leftrightarrow \boldsymbol{p}^e = \boldsymbol{p}^e \boldsymbol{A} + w \boldsymbol{l} + \frac{1-\gamma}{\gamma} w \boldsymbol{l}. \tag{2.1.9}$$

(2.1.9) shows that prices are made up of three components: material costs $\boldsymbol{p}^e \boldsymbol{A}$, labour costs $w\boldsymbol{l}$ and profits $\frac{1-\gamma}{\gamma} w \boldsymbol{l}$. Defining the wage-profit rate $e := \frac{1-\gamma}{\gamma}$ we get

$$\boldsymbol{\pi}^e = e w \boldsymbol{l}, \tag{2.1.10}$$

$$\boldsymbol{p}^e = \boldsymbol{p}^e \boldsymbol{A} + w \boldsymbol{l}(1+e), \tag{2.1.11}$$

$$\Leftrightarrow \boldsymbol{p}^e = w \boldsymbol{l} \left(\boldsymbol{I} - \boldsymbol{A}\right)^{-1} (1+e) = w \boldsymbol{\lambda} (1+e). \tag{2.1.12}$$

In that case, $w\boldsymbol{\lambda}$ can be interpreted as "monetary labour values" or "direct prices". These are prices directly proportional to labour values (Ochoa 1989, p. 416). Denoting them by $\boldsymbol{\delta}$ leads us to

$$\boldsymbol{p}^e = \boldsymbol{\delta}(1+e). \tag{2.1.13}$$

Thus, deviations from prices to direct prices are given by $w = \gamma w^*$, i.e. the workers share of nominal net product. The greater this share is, the lower deviations are. In other words, if profit is zero, (2.1.12) is equivalent to (2.1.5). (Note that w and e are globally defined because $\boldsymbol{b}$ is the same for all workers.) However, exchange ratios are not affected by this consideration because calculating relative prices and recalling (2.1.12) always yields:

$$\frac{p_i^e}{p_j^e} = \frac{\lambda_i}{\lambda_j}\ , \ i \neq j \text{ and } i,j = 1,\ldots,n. \tag{2.1.14}$$

Equation (2.1.14) now gives us the exact meaning of the famous phrase "law of value": *Labour values, i.e. direct and indirect labour time socially necessary to produce a commodity, are conserved in the exchange of commodities* (see Cockshott & Cottrell 1997, p. 545).
There are

$$\tau = \binom{n}{2} = \frac{n(n-1)}{2} \tag{2.1.15}$$

relative prices. The same applies to relative labour values. For notational convenience, we will call them $\boldsymbol{\rho}^e = (\rho_1^e, \ldots, \rho_\tau^e)$ and $\boldsymbol{\vartheta} = (\vartheta_1, \ldots, \vartheta_\tau)$, respectively. Now (2.1.14) becomes

$$\boldsymbol{\rho}^e = \boldsymbol{\vartheta}. \tag{2.1.16}$$

One problem remains. The derivation of (2.1.12) and (2.1.16) is based on (2.1.10) implying that profits are proportional to direct labour. Since this is equivalent to profit rates negatively connected to capital intensity we are dealing with the simple labour theory of value from "Capital, Volume I and II" (Marx 2001, 1972). The phrase "simple" means besides any considerations of transformation problem.

The question wether differences in sectoral capital intensity are disrupting the law of value leads us to our next section.

2.2 Neoricardian theory

Because of our last statement, neoricardian authors refuse, among other things[1], profit determination by (2.1.10) preferring

$$\boldsymbol{\pi}^n = r\boldsymbol{p}^n\boldsymbol{A} \tag{2.2.1}$$

instead. The superscript "n" stands for "neoricardian" and the scalar r indicates the uniform profit rate which is the equilibrium criterion in this approach.

$$\boldsymbol{p}^n = \boldsymbol{p}^n\boldsymbol{A}(1+r) + w\boldsymbol{l}, \tag{2.2.2}$$

$$\Leftrightarrow \boldsymbol{p}^n = w\boldsymbol{l}\left(\boldsymbol{I} - (1+r)\boldsymbol{A}\right)^{-1}. \tag{2.2.3}$$

Unlike the procedure in marxian economics (see (2.1.7)), neoricardian theorists do not fix w by assuming a wage basket. Instead, there are two income parameters w and r usually treating the latter as being exogenous. Expressing prices by an arbitrary $(n \times 1)$-commodity vector $\boldsymbol{d}$ we get

$$\boldsymbol{p}^n = w\boldsymbol{l}\left(\boldsymbol{I} - (1+\bar{r})\boldsymbol{A}\right)^{-1}, \tag{2.2.4}$$

$$\boldsymbol{p}^n\boldsymbol{d} = 1, \tag{2.2.5}$$

$$r = \bar{r}\ ,\ 0 < \bar{r} < r^*, \tag{2.2.6}$$

where r^* refers to the profit rate in case of zero wages. Now let $\boldsymbol{\eta} = (\eta_1, \ldots, \eta_\tau)$ be the vector or relative neoricardian prices. Similar to (2.1.16) we may write

$$\boldsymbol{\rho}^n = \boldsymbol{\eta}, \tag{2.2.7}$$

which could be viewed as "neoricardian price law". Comparing (2.1.12) to (2.2.4) we can see that in this case in general the law of value is not fulfilled. According to neoricardian framework there are only two exceptions for simple labour theory of value to hold:

$$\boldsymbol{\rho}^n = \boldsymbol{\vartheta} \begin{cases} \text{if} \quad r = 0, \\ \text{if} \quad \boldsymbol{lA} = \phi\boldsymbol{l}, \end{cases} \tag{2.2.8}$$

that is, zero profits or equal "organic composition of capital" which means that capital intensity is uniform (Kurz & Salvadori 1997, pp. 110–113, 120). Both conditions are not compatible to real capitalist economics. In this view, therefore, simple labour theory of value is a rather strange special case of neoricardian price theory. Hence, in reality there have to be significant deviations from prices to values according to differences in sectoral capital advanced.

[1] See Pasinetti (1977) or Kurz & Salvadori (1997) for details.

2.3 Decomposing prices

On the other hand, decomposing an arbitrary price system into profits and wages shows that these deviations are likely to be quite small (Shaikh 1984, pp. 64–68). To reproduce the argument let us go back to equation (2.1.9). There we have seen that prices are simply the sum of corresponding wage bill, profit and material costs. Now we use this statement without any assumption about profit determination such as (2.1.10) or (2.2.1):

$$\boldsymbol{p} = \boldsymbol{p}\boldsymbol{A} + w\boldsymbol{l} + \boldsymbol{\pi}. \tag{2.3.1}$$

Solving (2.3.1) for $\boldsymbol{p}$ provides

$$\boldsymbol{p} = w\boldsymbol{l}\left(\boldsymbol{I} - \boldsymbol{A}\right)^{-1} + \boldsymbol{\pi}\left(\boldsymbol{I} - \boldsymbol{A}\right)^{-1} = \boldsymbol{\delta} + \boldsymbol{\theta}, \tag{2.3.2}$$

where

$$\boldsymbol{\delta} := w\boldsymbol{l}\left(\boldsymbol{I} - \boldsymbol{A}\right)^{-1}, \boldsymbol{\theta} := \boldsymbol{\pi}\left(\boldsymbol{I} - \boldsymbol{A}\right)^{-1}. \tag{2.3.3}$$

Thus, any arbitrary price is made up by two components: Integrated labour costs, i.e. direct prices and integrated profits. After rearranging (2.3.2) and some algebraic manipulation we get:

$$\boldsymbol{p} = \boldsymbol{\delta}\left(\boldsymbol{I} + \frac{1}{w}\boldsymbol{\Lambda}^{-1}\boldsymbol{\Theta}\right), \tag{2.3.4}$$

with

$$\boldsymbol{\Lambda} := \mathrm{diag}(\lambda_1, \ldots, \lambda_n), \boldsymbol{\Theta} := \mathrm{diag}(\theta_1, \ldots, \theta_n). \tag{2.3.5}$$

Here, $\frac{1}{w}\boldsymbol{\Lambda}^{-1}\boldsymbol{\Theta}$ is the $(n \times n)$-diagonal matrix of integrated wage-profit rates. Its i-th element is a convex combination of profit-wage ratios that enters sector i via direct or indirect means of production (Shaikh 1984, p. 68). Again, we are interested in relative prices:

$$\frac{p_i}{p_j} = \frac{\delta_i\left(1 + \dfrac{\theta_i}{\delta_i}\right)}{\delta_j\left(1 + \dfrac{\theta_j}{\delta_j}\right)} = \frac{\lambda_i\left(1 + \dfrac{\theta_i}{\delta_i}\right)}{\lambda_j\left(1 + \dfrac{\theta_j}{\delta_j}\right)}, \; i \neq j \text{ and } i, j = 1, \ldots, n. \tag{2.3.6}$$

To facilitate analysis, we define

$$\boldsymbol{Z} := \operatorname{diag}(z_1, \ldots, z_\tau), \text{ where } z_k := \frac{1 + \dfrac{\theta_i}{\delta_i}}{1 + \dfrac{\theta_j}{\delta_j}}, \; k = 1, \ldots, \tau. \tag{2.3.7}$$

Comparing (2.1.14) to (2.3.6) we can see that (2.1.16) becomes

$$\boldsymbol{\rho} = \boldsymbol{\vartheta} \boldsymbol{Z}. \tag{2.3.8}$$

Now it is very important to recognize that all elements of $\boldsymbol{Z}$ are likely to be rather small because they depend on the degree of which different convex combinations of direct profit-wage ratios differ from each other. As a consequence, even large variations in sectoral profit-wage rates are reduced to small ones in corresponding integrated ratios. Therefore, (2.3.8) is a *modified law of value* with $\boldsymbol{Z}$ containing some kind of probably negligible disturbance factors.

But there is something more worth knowing about $\boldsymbol{Z}$. To see what it is we're bringing in capital stocks similar to (2.3.5).

$$\boldsymbol{k} = \boldsymbol{c} \left(\boldsymbol{I} - \boldsymbol{A} \right)^{-1}, \tag{2.3.9}$$

where now $\boldsymbol{c}$ denotes the $(1 \times n)$-vector of capital stocks, i.e. capital advanced at beginning of the uniform production period. Clearly,

$$\boldsymbol{\Theta} = r\boldsymbol{K}, \boldsymbol{K} := \operatorname{diag}(k_1, \ldots, k_n). \tag{2.3.10}$$

Hence, we can rewrite:

$$\frac{1}{w}\boldsymbol{\Lambda}^{-1}\boldsymbol{\Theta} = \frac{1}{w}\boldsymbol{\Lambda}^{-1} r\boldsymbol{K} = \frac{r}{w}\boldsymbol{\Lambda}^{-1}\boldsymbol{K}, \tag{2.3.11}$$

$$z_k := \frac{1 + \dfrac{\theta_i}{\delta_i}}{1 + \dfrac{\theta_j}{\delta_j}} = \frac{1 + \dfrac{rk_i}{w\delta_i}}{1 + \dfrac{rk_j}{w\delta_j}}, \; k = 1, \ldots, \tau. \tag{2.3.12}$$

(2.3.11) and (2.3.12) show that integrated profit-wage rates are proportional to integrated capital-labour ratios. Therefore, the above mentioned statement about the former also applies to the latter: even large variations in direct sectoral capital-labour-ratios are reduced to small variations in integrated ratios $\frac{r}{w}\boldsymbol{\Lambda}^{-1}\boldsymbol{K}$. Again, if there is any transformation problem, it is most likely moderate. But this is an empirical question.

2.4 Probabilistic Political Economy

In the probabilistic approach developed by Farjoun & Machover (1983), all variables such as prices, labour values, profit rates etc. are random ones. In place of analyzing a deterministic system with “mechanical” equilibrium properties like traditional marxian or neoricardian theorists do, they scrutinize the elements of an economic system similar to the way the behaviour of ideal gas molecules enclosed in a container is described by statistical mechanics (Farjoun & Machover 1983, pp. 39–56). In their view, the transformation problem occurs because of using an inappropriate concept of equilibrium, namely the adoption of a uniform profit rate (Farjoun & Machover 1983, pp. 28–38). Instead, they suppose profit rates to be described by a gamma distribution and replace the assumption of equalizing profit rates by the more sophisticated assertion that for a given country in a state of equilibrium the probability density function (pdf) of profit rates is virtually independent of time (Farjoun & Machover 1983, pp. 64–66).

Remarkably, this procedure results in relationships analog to section 2.1, that is simple labour theory of value probably holds in spite of heterogeneous capital intensity. We should give a brief survey of the relevant proceeding.

First, define the *specific price* of commodity i as follows:

$$\Psi_{i_{FM}} := \frac{p_i}{\lambda_i}, i = 1, \dots, n. \tag{2.4.1}$$

In terms of Adam Smith, $\Psi_{i_{FM}}$ can be interpreted as ratio of labour commanded to labour embodied. Surely, it cannot generally be less than one for then the selling price of a commodity does not even meet its direct and indirect wage costs. Furthermore, if $\Psi_{i_{FM}}$ would not be a random variable but degenerate at unity, we would fall back to our introductory world without capitalists (equation (2.1.3) and (2.1.4)).

Now look back on (2.3.2). It says that prices are made of integrated labour costs and integrated profits. In that case, the price of commodity i is

$$p_i = \delta_i + \theta_i, i = 1, \dots, n. \tag{2.4.2}$$

Dividing by the i-th labour value yields

$$\Psi_{i_{FM}} = \frac{\delta_i}{\lambda_i} + \frac{\theta_i}{\lambda_i}, \tag{2.4.3}$$

with expected value

$$E\left(\Psi_{FM}\right) = E\left(\frac{\delta}{\lambda}\right) + E\left(\frac{\theta}{\lambda}\right). \tag{2.4.4}$$

Since

$$E\left(\frac{\delta}{\lambda}\right) = \sum \alpha_i \left(\frac{\delta_i}{\lambda_i}\right), \text{with weights } \alpha_i = \frac{\lambda_i}{\sum \lambda_i}, i = 1, \ldots, n, \tag{2.4.5}$$

we obtain

$$E\left(\frac{\delta}{\lambda}\right) = \frac{\sum \delta_i}{\sum \lambda_i} = E(w), \tag{2.4.6}$$

and similarly,

$$E\left(\frac{\theta}{\lambda}\right) = \frac{\sum \theta_i}{\sum \lambda_i} = e^* E\left(\frac{\delta}{\lambda}\right), \text{where } e^* = \frac{\sum \theta_i}{\sum \delta_i}. \tag{2.4.7}$$

Hence,

$$E\left(\Psi_{FM}\right) = (1 + e^*) E\left(w\right). \tag{2.4.8}$$

If we define the average hourly wage $E(w)$ as unit of account, i.e. $E(w) = 1$, (2.4.8) reduces to

$$E\left(\Psi_{FM}\right) = 1 + e^*. \tag{2.4.9}$$

For that reason, $E(\Psi_{FM})$ depends on the global wage-profit rate. But this means that (2.4.9) is the stochastically counterpart to (2.1.12). To put it precisely: If we displace the assumption of uniform profit rates by considering the pdf instead, simple labour theory of value holds as a *statistical law*, even in a state of equilibrium.

To concretize further discussion, Farjoun & Machover assumed Ψ_{FM} to be described by the following normal distribution:

$$\mathcal{N}(1 + e^*; \sigma) = \mathcal{N}(2; 1/3). \tag{2.4.10}$$

The authors make this suggestion because they believe that in developed capitalist countries $e^* = 1$, at least approximately (which in fact is not true). Furthermore, in their view standard deviation σ should be $1/3$ because in this case the probability of $\Psi_{i_{FM}} < 1$ would be less than $1/1000$ which they suppose to be quite realistic.

3 Empirical framework

3.1 Econometrics

Before explaining regression details it is recommended to recall the modified law of value:

$$\boldsymbol{\rho} = \boldsymbol{\vartheta}\boldsymbol{Z}. \tag{2.3.8}$$

In case of output level unity we may use the natural logs. (Note that all elements in (2.3.8) are dimensionless quantities in that case.) This yields

$$\tilde{\boldsymbol{\rho}} = \tilde{\boldsymbol{\vartheta}} + \tilde{\boldsymbol{z}}, \tag{3.1.1}$$

where the tilde ($\tilde{}$) denotes the natural logarithm, i.e. $\tilde{\boldsymbol{\rho}} = (\ln(\rho_1), \dots, \ln(\rho_r))$ etc. and $(1 \times n)$-vector $\boldsymbol{z}$ contains the diagonal elements of $\boldsymbol{Z}$. Thus, an evident way to test the aforementioned theories is to perform the corresponding double-log regression model

$$\tilde{\boldsymbol{\rho}}^T = \boldsymbol{\Delta}\boldsymbol{\beta} + \boldsymbol{\epsilon}, \tag{3.1.2}$$

with column vectors of regression parameters $\boldsymbol{\beta}$ and error terms $\boldsymbol{\epsilon}$. We provide four econometric models:

$$\text{Model L1:} \quad \boldsymbol{\Delta} = \begin{pmatrix} 1 & \tilde{\vartheta}_1 \\ \vdots & \vdots \\ 1 & \tilde{\vartheta}_\tau \end{pmatrix}, \; \boldsymbol{\beta} = \begin{pmatrix} \beta_0 \\ \beta_1 \end{pmatrix}. \tag{3.1.3}$$

Labour values are calculated by assuming a common dummy wage rate $w = 1\,€/h$ such that $\boldsymbol{\delta} = \boldsymbol{\lambda}$. This implies that inter-sectoral wage differentials are caused directly by different skill levels. Or, rephrased, skilled labour is expressed in units of simple labour (Cockshott & Cottrell 1997, p. 546).

$$\text{Model L2:} \quad \boldsymbol{\Delta} = \begin{pmatrix} 1 & \tilde{\vartheta}_1 & \tilde{z}_1 \\ \vdots & \vdots & \vdots \\ 1 & \tilde{\vartheta}_\tau & \tilde{z}_\tau \end{pmatrix}, \; \boldsymbol{\beta} = \begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{pmatrix}, \tag{3.1.4}$$

$$\text{Model L3:} \quad \boldsymbol{\Delta} = \begin{pmatrix} 1 & \tilde{\vartheta}_1 & \tilde{s}_1 & \tilde{u}_1 \\ \vdots & \vdots & \vdots & \vdots \\ 1 & \tilde{\vartheta}_\tau & \tilde{s}_\tau & \tilde{u}_\tau \end{pmatrix}, \; \boldsymbol{\beta} = \begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix}. \tag{3.1.5}$$

Model L2 is self-explanatory. Model L3 will be used to detect factors of influences that maybe are misleadingly excluded by theory, namely relative direct wage-profit rates and relative capital intensity:

$$s_k := \frac{1 + e_i}{1 + e_j}, \; k = 1, \ldots, \tau, i \neq j \text{ and } i, j = 1, \ldots, n, \tag{3.1.6}$$

$$u_k := \frac{\dfrac{c_i}{w_i l_i}}{\dfrac{c_j}{w_j l_j}}, \; k = 1, \ldots, \tau, i \neq j \text{ and } i, j = 1, \ldots, n. \tag{3.1.7}$$

Finally,

$$\text{Model N:} \quad \boldsymbol{\Delta} = \begin{pmatrix} 1 & \tilde{\eta}_1 \\ \vdots & \vdots \\ 1 & \tilde{\eta}_\tau \end{pmatrix}, \; \boldsymbol{\beta} = \begin{pmatrix} \beta_0 \\ \beta_1 \end{pmatrix}. \tag{3.1.8}$$

Model N, in turn, requires some comment. In (2.2.3), $\boldsymbol{A}$ is based on flow terms. But in reality, obviously, there are stocks, too. Actually, this would require to add a matrix of capital coefficients and to calculate r on a stock basis. Unfortunately, in case of German data this matrix is not available. There is only knowledge about the money value of sectoral capital stocks $\boldsymbol{c}$. Therefore, the following procedure is made: neoricardian prices are computed despite of lacking capital coefficient but by using money value of sectoral capital stocks for calculating r. In doing so, another crucial point occurs: Applying stocks depends on defining turnover times. Because sectors in Input-Output (IO) tables include a broad mix of different production periods, this is a serious problem which is hardly to handle in a satisfying way (Tsoulfidis & Maniatis 2002, pp. 368–369). Thus, two polar assumptions can be introduced to make regression analysis possible: First, individual time differences effectively cancel out. Profit rates then should only base on $\boldsymbol{c}$. Second, the production period takes one year as it is implemented in national accounts. In this case, r should be better estimated with respect to $\boldsymbol{c}$ and sectoral inputs known from $\boldsymbol{A}$, too. Probably, the truth lies somewhere in the middle. But as a matter of fact, both possibilities lead to almost the same regression results, so there is no need to be worried about these things too much. Instead, it is appropriate to choose this method whose fit is (marginally) better. Therefore, for pragmatical reasons, we suppose the uniform production period to be one year.

In addition, Farjoun's and Machover's statements (2.4.9) and (2.4.10) should be checked as well as their claim of gamma distributed profit rates.

3.2 Data

Data is taken from the German Federal Bureau of Statistics which offers IO tables including information on 71 sectors. Because statistics on German capital stocks only contain 55 sectors, the relevant columns and rows of IO tables have to be merged such that every sector meets a figure from capital stocks.[1] Current year is 2004.

Since labour theory of value implies the distinction of productive and unproductive labour, the following rows are treated as being surplus value: Finance, assurance, real estate, educational and social services including all other kind of public or non-commercial services.[2] Moreover, taxes are taken as being profits and sectoral outputs are evaluated at producer prices to avoid confusion caused by trade margins (Shaikh & Tonak 1994, pp. 78–81).

Furthermore, some sectors were removed from regression analysis because there are outliers inducing non-normal error terms. This procedure is harmless since all of these sectors are either highly state-regulated (coal, water supply), rent-biased (oil) or offer non-market goods. After all, there remain 38 sectors. Hence, $\tau = \frac{n(n-1)}{2} = 703$.

3.3 Criticism

There are mainly two arguments to disbelieve empirical work on price-value deviations. First of all, as Kliman (2002, 2005) put it, any correlation between labour values and market prices may be spurious as long as we do not deflate them by sectoral size, i.e. sectoral costs. He finds out that correlations vanished under this procedure. Díaz & Osuna (2005–06) argue the same way. Cockshott & Cottrell (2005) discuss this point claiming that (1) the sector size is irrelevant since in IO tables (physical) unit size could not be appropriately defined and (2) the deflating method used by Kliman, though theoretically harmless, must destroy correlations in practice even if they are true. This is because computing labour values is based on IO tables which are made up of costs. Thus, costs are a source data of labour values. Now, if we deflate sectors by costs we eliminate the source data on labour values leaving just an error term. That is, a vanishing correlation is not surprising but caused by a special choice of method.

The second point of critique is quite fundamental. Díaz & Osuna (2007)

[1]Detailed information on measuring capital stocks in Germany can be found in Schmalwasser & Schidlowski (2006).

[2]This terminology is rather misleading. It would be more precise to speak of surplus-creating labour and surplus-consuming labour instead. Shaikh & Tonak (1994, pp. 20–32, 74) and Mohun (2003) give further explanations.

state that *every* cross sectional price-value correlation is spurious, regardless of industrial size and deflating method, because regression results depend on the unit of measurement.[1] To follow their argument, we have to recall the law of value (2.1.14). It is defined per unit output. In practice, using IO tables, it becomes:

$$\frac{p_i x_i}{p_j x_j} = \frac{\lambda_i x_i}{\lambda_j x_j}, \tag{3.3.1}$$

where x_i denotes the quantity of commodity $i, i = 1, \ldots, n$. The corresponding k-th regression equation is

$$\ln\left(\frac{p_i x_i}{p_j x_j}\right) = \beta_0 + \beta_1 \ln\left(\frac{\lambda_i x_i}{\lambda_j x_j}\right) + \epsilon_k, k = 1 \ldots, \tau. \tag{3.3.2}$$

Díaz & Osuna (2007, p. 392) present (3.3.2) as follows:

$$\begin{aligned} \ln\left(\frac{p_i}{p_j}\right) + \ln\left(\frac{x_i}{x_j}\right) = \beta_0 + \beta_1 \ln\left(\frac{\lambda_i}{\lambda_j}\right) + \ln\left(\frac{x_i}{x_j}\right) \\ + \beta_1 \ln\left(\frac{x_i}{x_j}\right) - \beta_1 \ln\left(\frac{x_i}{x_j}\right) + \epsilon_k. \end{aligned} \tag{3.3.3}$$

By rearranging they yield

$$\ln\left(\frac{p_i x_i}{p_j x_j}\right) = \beta_0 + \beta_1 \ln\left(\frac{\lambda_i x_i}{\lambda_j x_j}\right) + (1 - \beta_1) \ln\left(\frac{x_i}{x_j}\right) + \epsilon_k. \tag{3.3.4}$$

Therefore, Díaz & Osuna claim that regression results depend on physical units in $\ln\left(\frac{x_i}{x_j}\right)$. In their view, this constitutes an unavoiding problem of indeterminacy in price-value correlation measures.

But why is there a difference between (3.3.2) and (3.3.4)? To understand the reason, one has to remember that arguments of transcendental functions are always dimensionless. Consider, for example, the expression e^t. Its Taylor expansion is $\mathrm{e}^t = 1 + t + \frac{t^2}{2!} + \frac{t^3}{3!} + \cdots$. Obviously, due to dimensional homogeneity, t must be a *pure number*. Because $\mathrm{e}^t = x$ and $\log(x) = t$, the same statement holds in case of logarithm (Szirtes 2007, pp. 104, 108).

[1] Steedman & Tomkins (1998) make a related objection but provide a solution, too. See Tsoulfidis & Maniatis (2002, pp. 365–366) as well. It is not necessary to discuss this issue in more detail because it is of no consequence for the following investigation.

Now let us have a look on the dimensions in (3.3.1) and (3.3.2). Examining expression (3.3.1) shows that all elements are dimensionless scalars. Hence, we may log-transform and write it like (3.3.2). Still, merely dimensionless scalars appear and, as a consequence, no dependance on units occurs. Yet, converting (3.3.2) into (3.3.3) and (3.3.4) destroys dimensional homogeneity. Relative prices should be added to relative quantities in that case. But this is impossible: "Apples can only be added to apples, not oranges." (Vignaux & Scott 1999, p. 32)

The mistake comes in because the authors disregard the fact that any logarithm is only defined in case of pure numbers. Given some quantities $a, b, \log(ab)$ may exist even though $\log(a)$ and $\log(b)$ do not (de Jong & Quade 1967, pp. 188–189). Hence, $\log(ab) = \log(a) + \log(b)$ is correct *if and only if* both a and b are *dimensionless quantities*. Equation (3.3.3) and (3.3.4) do not fulfill this prerequisite. In fact, facing unit dependencies is essentially a strong hint on dimensional heterogeneity. Or, rephrased, every equation which is dimensionally homogenous is formally independent of the choice of units (de Jong & Quade 1967, p. 28). Therefore, Díaz's and Osuna's criticism provides no argument for deciding whether price-value correlations are spurious or not.

4 Results

4.1 Price-value deviations

Table 2 shows regression results of model L1, L2, L3 and N. Because of heteroscedastic and autocorrelated error terms (see Table 1) they were fitted by Generalized Least Squares (GLS) instead of using Ordinary Least Squares (OLS).[1] Note that all other empirical studies on price-value deviations do not ensure the usual OLS assumptions to be fulfilled. Therefore, the results reported in previous studies might be overestimated. In this case, considering correlation coefficients or coefficients of determination do not make sense (see, for instance, Ramanathan (2002, pp. 347, 385)). But also note that several appropriate criterions show for all models a high goodness-of-fit as well. These

[1] Autocorrelation was detected by plotting the residuals' partial autocorrelation functions (pacf). Thus, applying an appropriate autoregressive (AR) model to them was necessary. See Shumway & Stoffer (2006, pp. 106–110, 293–295). Using a Breusch-Pagan test shows heteroscedastic residuals in case of Model L3. We chose the software R for running regression analysis. Its package nmle deals with several options to handle such kind of error problems. Pinheiro & Bates (2000, chapter 5) provide further information. The corresponding software output is listed in the appendix.

Table 1: OLS error problems

	L1	L2	L3	N
Autocorrelation:				
Residuals pacf plot	AR(18)	AR(18)	AR(13)	AR(18)
Heteroscedasticity:				
Breusch-Pagan test	1.065	5.761	7.613	5.728
p-value	0.302	0.016	0.006	0.017

Table 2: GLS fit (maximum likelihood) of L1, L2, L3 and N, 2004.

	L1	L2	L3	N
β_0	−0.020	−0.023	−0.039 *	0.012
β_1	0.924 ***	0.900 ***	1.000 ***	0.924 ***
β_2	—	−0.043	0.548 ***	—
β_3	—	—	−0.015 ***	—
AIC	−409.204	−412.806	−1821.788	−390.156
BIC	−313.541	−308.033	−1735.236	−289.938
logLik	225.602	229.403	929.894	217.078

Signif. codes: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

are Akaike’s information criterion (AIC), Bayesian information criterion (BIC) and the log-likelihood (LogLik) of estimations.

Remarkably, labour theory of value and neoricardian theory show nearly identical results. This is much in line with previous results (Cockshott et al. (1995, p. 107); Tsoulfidis & Maniatis (2002, p. 361)), although in the first mentioned study L1’s elasticity of labour values is more closely to one, which is clearly the theoretical ideal (Table 3). It should be noted that these studies do not take capital stocks into account. Besides methodological differences, the result of L3 may show us why β_1 in L1 is significantly less than in previous work. This could be the case because s has a noticeable influence on prices – we will come back to this point in the next section. Remember that by choice of method s does not reflect different skill levels. An exogenous explanation might be that labour markets are not perfectly competitive as it is implicitly assumed in marxian theory. Maybe this leads to a greater dispersion of s, hence causing lower values of β_1 in L1. Moreover, there is no meaningful difference between L1 and L2 in explaining relative prices.[1] Therefore, as supposed in 2.3, the influence of disturbance elements in $\boldsymbol{Z}$ is negligible. While evaluating model N recall our pragmatical assumption on production time. If we compute profit rate without supposing uniform production period to be one year, the estimation of β_1 does not change crucially but AIC, BIC and logLik significantly decrease. All in all, Shaikh's explanations are impressively confirmed.

Now consider the distribution of deviations from relative prices to relative values and relative prices to relative neoricardian prices. We denote them Ψ and Φ. Moreover, remember direct prices (2.4.1) which we have noted Ψ_{FM} to avoid confusion.

$$\Psi_k := \frac{\rho_k}{\vartheta_k}, k = 1, \ldots, \tau, \tag{4.1.1}$$

$$\Phi_k := \frac{\rho_k}{\eta_k}, k = 1, \ldots, \tau, \tag{4.1.2}$$

$$\Psi_{i_{FM}} := \frac{p_i}{\lambda_i}, i = 1, \ldots, n. \tag{4.1.3}$$

Table 4 gives an overview about distributional characteristics. We can see that differences between Ψ and Φ are not significant. Both means are factual 1 and their standard deviations are about 2.8. This is quite narrow since the same applies to coefficients of variation, too. Thus, labour values and neoricardian prices both show almost ideal results.

But Farjoun's and Machover's statement (2.4.10) does not hold exactly. They maintained that

$$E\left(\Psi_{FM}\right) = 1 + e^*. \tag{2.4.9}$$

Since $e^* = 0.788$ this implies

$$1.988 \approx 1 + 0.788 = 1.788. \tag{4.1.4}$$

which is an acceptable prediction. But, more important, Ψ_{FM} is rather log-normal distributed than normal distributed. The same holds in case of Ψ and Φ. The reason becomes clear by remembering (3.1.1), i.e. influences on prices are multiplikative, not additive. We can see this in figures 1, 2 and 3 showing both

[1] Actually, the negative sign of β_2 in L2 is a hint on the problem of multicollinearity. Further analysis confirms this suspicion. However, since model L2 do not provide improved estimations compared to model L1 it is not necessary to go into details.

Table 3: Regression outcomes in different countries.

	Coef.	L1	N
Greece 1970	β_0	2032	267.0
	β_1	1.15	0.979
UK 1984	β_0	-0.055	-0.049
	β_1	1.014	1.024
Germany 2004	β_0	-0.020	0.012
	β_1	0.924	0.924

Source: Cockshott et al. (1995, p. 107) (UK), Tsoulfidis & Maniatis (2002, p. 361) (Greece).
Notes: All estimations of β_1 are significant at the 1% level.

Table 4: Summary statistics of Ψ, Ψ_{FM} and Φ, 2004.

	Ψ	Ψ_{FM}	Φ
Mean	0.967	1.988	0.997
Median	0.935	1.887	0.970
Standard deviation	0.278	0.444	0.284
Coefficient of variation	0.288	0.223	0.285

the relevant histograms and pdf estimations. The latter are derived by taking the mean and standard deviations of $\ln(\Psi)$, $\ln(\Phi)$ and $\ln(\Psi_{FM})$. Applying a Jarque-Bera test shows that all of them are likely to be normally distributed. Afterwards, Kolmogorov-Smirnov (KS) test was used to check if the resulting log-normal distribution meets the data.[1] From there it is known that Ψ could be described by the log-normal distribution Log-$\mathcal{N}(-0.07; 0.29)$. Similarly, $\Phi \sim$ Log-$\mathcal{N}(-0.04; 0.29)$. In terms of theory, this is marginally better. We can use the estimated pdf and calculate expected values of deviations. This yields $E(\Psi) = 1.07$ and $E(\Phi) = 1.10$. At least, $\Psi_{FM} \sim$ Log-$\mathcal{N}(0.66; 0.21)$ which contradicts (2.4.10). This implies $E(\Psi_{FM}) = 2.15$.

It is quite remarkable how well simple labour theory of value is in line with empiricism. Though the same holds for neoricardian theory, this is not

[1] While applying a KS test the underlying distribution parameters usually needs to be theoretically specified (Ricci 2005, p. 19). To avoid this problem a Monte-Carlo-based version was used instead (see Wolter 2008, p. 11, for details).

Figure 1: Histogram and pdf of Ψ, 2004.

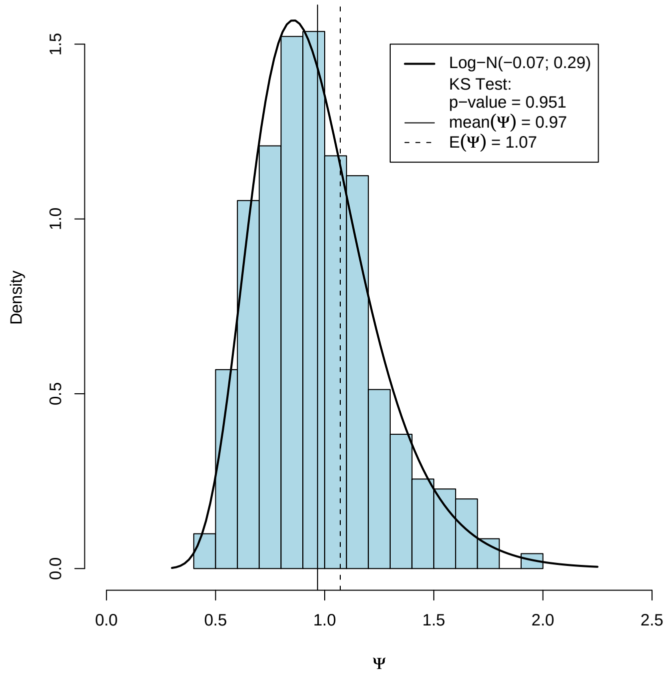


Figure 2: Histogram and pdf of Φ, 2004.

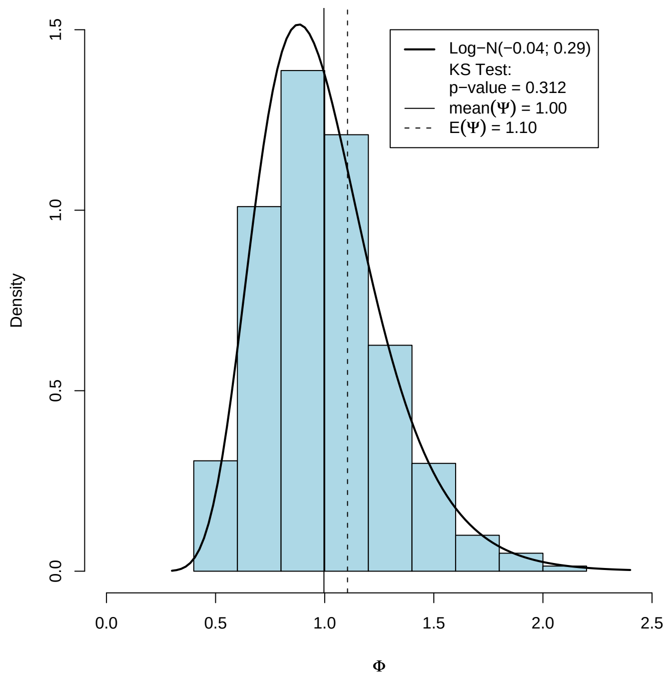


Figure 3: Histogram and pdf of Ψ_{FM}, 2004.

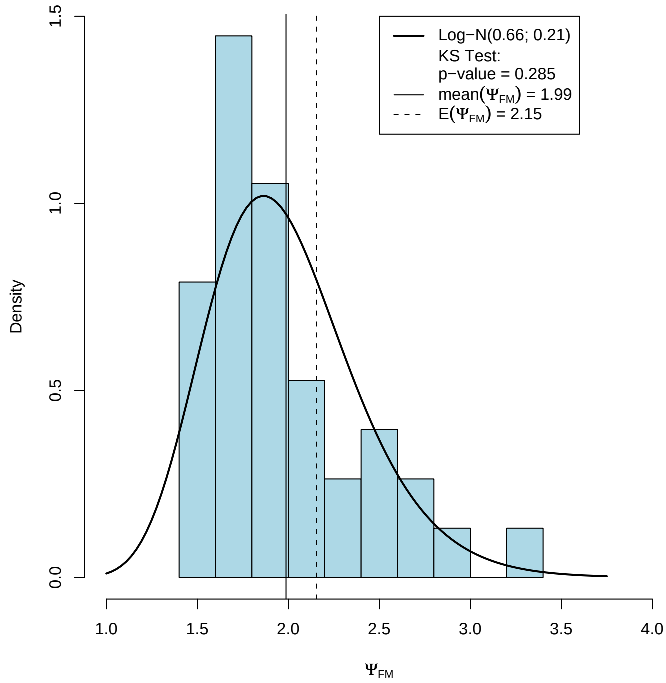


Figure 4: Predicted and factual pdf of Ψ_{FM}, 2004.

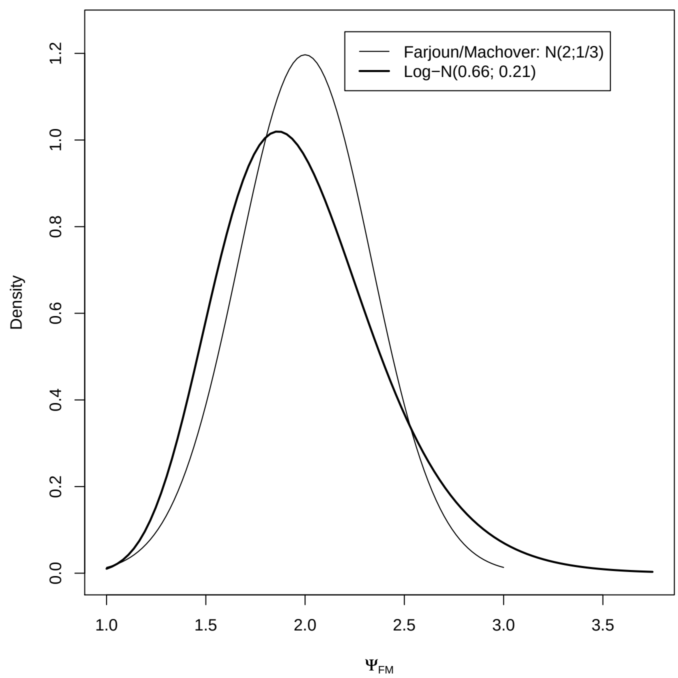


Table 5: Summary statistics of r, q and e, 2004.

	r	q	e
Mean	0.137	12.177	1.187
Median	0.108	7.100	0.848
Standard deviation	0.090	14.526	1.024
Coefficient of variation	0.659	1.193	0.862

very surprising since nearly the whole body of both marxian and neoricardian literature supposed it to be state of the art. But hardly anybody of the theorists expects *both* theories to fit the data. Anyway, on the basis of section 2.3 and 2.4 there are good reasons to be not far too surprised.

Neoricardians might argue that there are indeed differences, however small, so neoricardian theory should be preferred. But this is not true. Because differences are negligible small, we should rather take Occam's razor and favour that theory which is less complex. For two reasons, this is simple labour theory of value: First, labour values can be computed without any need for data on capital stocks or capital coefficients. Second, we do not have to struggle with production periods because labour values do not depend on profit rates. Thus, using neoricardian prices for empirical research is more error-prone than applying labour values.

4.2 The economics of profit rates and surplus rates

We continue our empirical study by analyzing profit rates and surplus rates. Capital intensity q may also be of interest, i.e. capital advanced divided by paid wages. Table 5 gives a summary statistic. Obviously, none of these variables have narrow distributions. In case of profit rates and capital intensity this is not amazing, whereas in marxian literature e's distribution often is supposed to be narrow. For instance, Farjoun & Machover (1983, pp. 32, 70) and Cockshott & Cottrell (1998, p. 77)[1] argued this way. In fact, the corresponding coefficient of variation is greater than the profit rate's is. But Farjoun's & Machover's claim of gamma distributed profit rates was well founded. As Figure 5 demonstrates, in 2004 we have $r \sim \Gamma(2.78; 20.29)$ with pretty goodness of fit. Moreover, their equilibrium assumption seems to be true as well. Calculating year 2000 profit rates yields $r \sim \Gamma(2.03; 15.65)$. Using a KS test shows that there is no

[1] Note that the authors define e in a different way.

Figure 5: Histogram and pdf of profit rate, 2004.

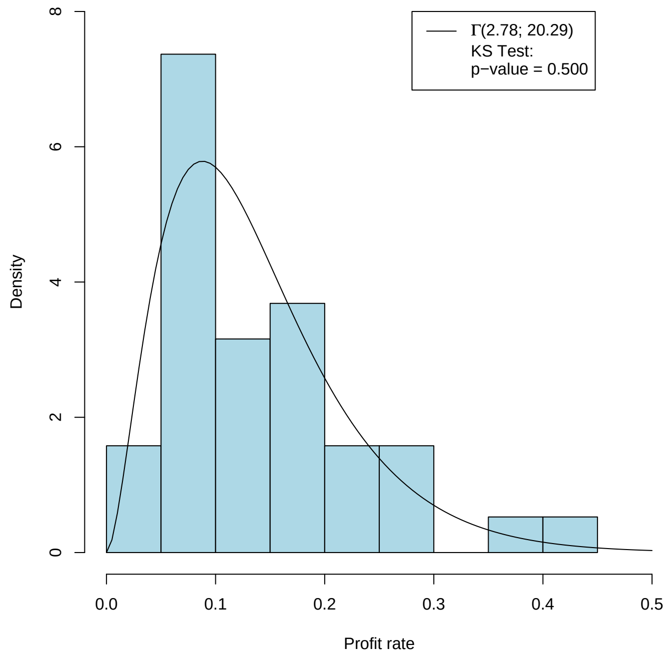


Figure 6: Profit rate equilibrium, 2000 and 2004.

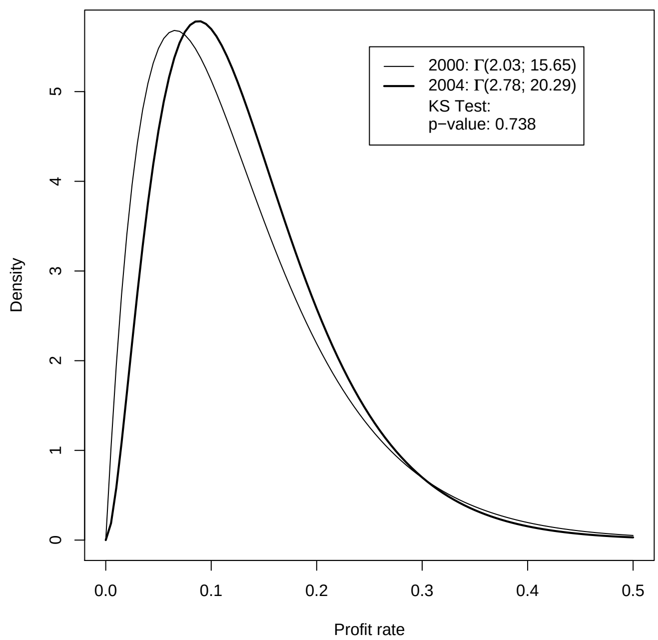


Table 6: Correlation matrix of $\tilde{r}$, $\tilde{q}$, $\tilde{e}$, $\tilde{\Psi}_{\text{FM}}$ and $\tilde{\Phi}_{\text{FM}}$, 2004.

	$\tilde{q}$	$\tilde{r}$	$\tilde{e}$	$\tilde{\Psi}_{\text{FM}}$	$\tilde{\Phi}_{\text{FM}}$
$\tilde{q}$	1.000				
$\tilde{r}$	−0.595	1.000			
$\tilde{e}$	0.684	0.179	1.000		
$\tilde{\Psi}_{\text{FM}}$	0.653	0.166	0.950	1.000	
$\tilde{\Phi}_{\text{FM}}$	0.450	0.304	0.827	0.896	1.000

Notes: For a sample size of 38, 5% and 1% critical values of correlation coefficients are $\approx |0.320|$ and $\approx |0.413|$, respectively. Φ_{FM} is defined similar to Ψ_{FM} in case of neoricardian prices of production.

significant difference to 2004 (see. Figure 6). Hence, it could be argued that there is no equalization tendency for profit rates as neoricardian authors state.

Now let us take functional relationship into account. Table 6 provides a correlation matrix for all relevant variables. Several points are of interest. First, and most important, there is a negative correlation between profit rate and capital intensity.[1] Figure 7 may clarify this issue. This is a very remarkable result first shown in Cockshott & Cottrell (2003). It challenges the whole body of literature on transformation problem and neoricardian theory because it is always taken for granted that profit rates must be independent of capital intensity. Only simple labour theory of value predicts this incidence and it is precisely due to this fact that it is usually thought to be fundamentally flawed.

On top of that, there is another unexpected positive correlation between wage-profit rate and capital intensity. This seems to be the counterpart to the negative relationship between $\tilde{r}$ and $\tilde{q}$ (see Figure 8 as well). Sectors producing with relatively high capital equipment per working hour partially compensate the comparatively lower profitability of capital stock by arranging an appropriate wage-profit rate. Hence, $\tilde{q}$ must have impact on $\tilde{\Psi}_{FM}$, too; but model L3 (see Table 2) shows that this influence is rather meaningsless if we take both $\tilde{s}$ and $\tilde{u}$ into account. Nevertheless, one might interpret this as a kind of transformation process.[2] On the basis of section 2.1, this is a

[1] As it is mentioned before, results do not depend on the assumption of a yearly production period. On the contrary, calculating r and q with respect only to capital stocks $\boldsymbol{c}$ increases the relevant correlation coefficients. Also note that Table 6 is based on log-transformed variables because the relationships are not linear.

[2] Cockshott & Cottrell (1998, p. 82) also recognized this outcome, but their results are less convincing because they did not take capital stocks into account.

Figure 7: Relation between capital intensity and profit rates, 2004.

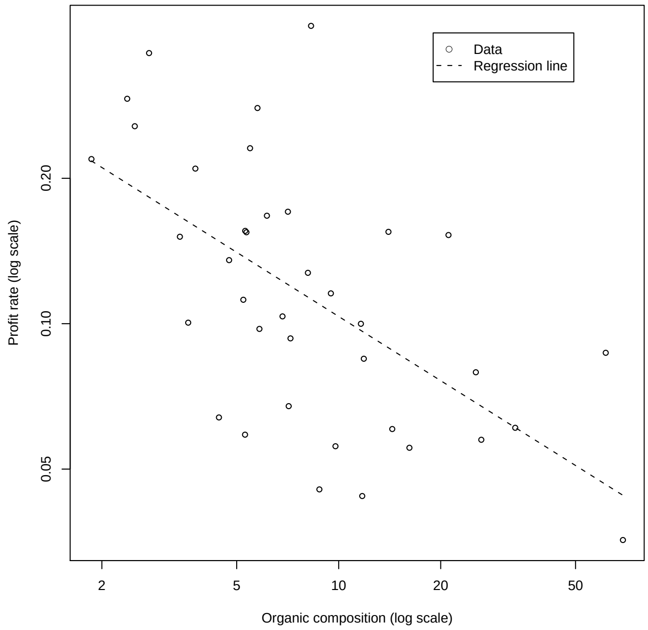
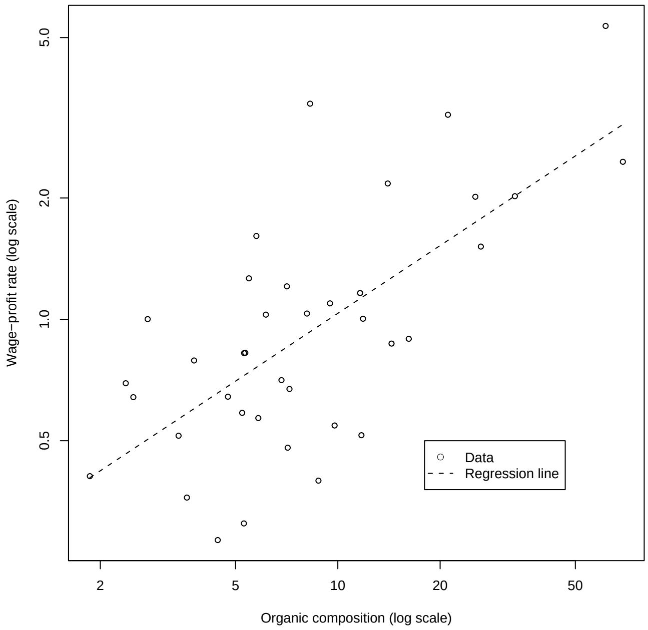


Figure 8: Relation between capital intensity and wage-profit rates, 2004.



serious problem because e should be uniform having no systemic influence on its own. In this respect, the deterministic or "mechanical" version of simple labour theory is effectively wrong. But reformulated as a kind of Probabilistic Political Economy, it is not affected. More important, though a certain transformation tendency appears, this phenomena is not strong enough to fully compensate the effects of different capital intensities. As a consequence, $\tilde{\Phi}_{FM}$ and $\tilde{q}$ are positively correlated as well. Obviously, this is a strong provocation for all theories based on the assumption of non-dependency between profit rate and capital intensity.

5 Conclusion

This paper gives similar results to those of the previous studies concerning labour values and market prices. It is argued that both simple labour theory of value and neoricardian theory yield very good results in explaining data. Differences in estimated outcomes are mainly negligible. Therefore, noticing the approaches developed by Shaikh (1984) and Farjoun & Machover (1983) and having Occam's razor in mind, we should prefer simple labour theory of value for analyzing real world phenomena. In addition, there is one critical point for neoricardian theory: The basis of the transformation debate seems to be wrong because profit rates and capital intensity are negatively correlated. Moreover, there are hints on gamma distributed profit rates which are therefore not uniform. The corresponding density functions do not change significantly during 2000 and 2004. Hence, in terms of Probabilistic Political Economy, it seems that the German economy was in a state of statistical equilibrium.

After all, although most marxian authors deny the relevance of labour values for explaining prices, there are good reasons to argue that the law of value is correct in a stochastic sense. Without going into detail, this implies that the famous marxian invariant postulates are justifiable in a similar way. Profit, therefore, is not based on the marginal product of capital but on exploited labour. But with due respect to the reader's resources, questions like this may be discussed some other time.

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