‘Neoricardian Theory and the Measurement of Prices of Production’: An Alternative Approach*

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Abstract

This paper investigates the empirical approach to the measurement of prices of production and wage-profit curves proposed by Han and Schefold (2006) in this Journal. A number of theoretical and empirical shortcomings of this approach are discussed, focusing in particular on the neglect of capital stock matrices and on the empirically objectionable assumption of uniform profit and wage rates. An alternative approach for the empirical analysis of wage-profit curves and prices of production is proposed and its main properties are investigated using a new dataset on the German economy (1991-1999). It is concluded that a Leontief-Bródy approach (augmented by profit rate differentials) is more appropriate for theoretical as well as applied analysis than the purely theoretical orientation that characterizes the Sraffa / von Neumann world.

Keywords: wage-profit curves, prices of production, fixed capital, input-output analysis, choice of technique

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1 Introduction

Capital theory is at the heart of theories of value and distribution, and indeed it has been at the center of substantial controversy among alternative approaches. Quite surprisingly, though, virtually all the contributions to the debate have focused on purely theoretical issues. In a recent issue of this journal, however, Han and Schefold (2006) have provided an innovative and thought-provoking empirical analysis of paradoxes in capital theory - such as reverse capital deepening and reswitching. Han and Schefold (2006) investigate data from nine OECD countries in the period (1986 – 1990). and construct thirty-two 36-sectoral input-output tables, and the corresponding vectors of labor inputs, from the OECD database. Although there are a number of fine points in their analysis that may be worth discussing, the main methodological innovation of Han and Schefold (2006) is not to consider data from one country at a certain point in time as a given set of data describing an organic economic system, but to implement pairwise comparisons of the thirty-two given data sets under the assumption that a given technique used in a country at a certain date is also available in principle in a different country at a different point in time. Thus, for example, they compare Germany 1990 and Canada 1990 and, when they are mixed together, this gives two processes for each industry and thus 2^{36} possible economy-wide techniques, or mixed input-output matrices. This gives them a very large amount of data for their analysis as there are $\binom{32}{2}$ pairs of input-output matrices. On the basis of this type of mixed input-output matrices, they calculate the envelopes of the wage-profit curves and investigate the occurrence of reverse capital deepening and reswitching.

Indeed, the novel methodology adopted by Han and Schefold (2006) raises important issues for the empirical and theoretical analysis of classical price theory, beyond the analysis of paradoxes in capital theory, as it provides a general framework for the measurement of prices of production and wage-profit curves. In this paper, we highlight a number of shortcomings of their approach, and provide an alternative framework for the analysis of prices of production and distributive conflict between capital and labor from a theoretical and an empirical viewpoint.

To be specific, in this paper, we analyze input-output data for the German economy (for the period 1991-1999), including the construction of a capital stock matrix, and argue that the approach of Han and Schefold (2006) is very misleading from the empirical perspective, since their results are based on wage-profit curves that are irrelevant for the analysis of the actual choice of technique. Firstly, and this is our central critique of their paper, the composition of processes from various countries and various times into an artificial intermediate input-output structure is not only of a very hypothetical nature, but it is also illegitimate if the capital stock matrices underlying the single processes are taken into account.

Secondly, based on these findings we use the input-output and capital stock matrices provided by Kalmbach et al. (2005),[1] appropriately extended by one of the

[1] See also Franke and Kalmbach (2005).

present authors, to show that there is no tendency towards a uniform rate of profit in the German economy over the period considered. We here follow Farjoun and Machover (1983) who claimed that the restriction of a Classical price theory to the consideration of a uniform rate of profit is utterly unrealistic. This holds in particular if sectoral disaggregation is very high, because the structure of input-output tables which is based on averaging procedures then starts to show the extent of heterogeneity that characterizes the production of a specific commodity in actual production processes.

Conventionally defined prices of production thus should give way to the introduction and consideration of sectoral profit and wage rate differentials, as well as capital stocks, and their implications for a theoretical and empirical analysis of technical change.[2] In this respect the paper makes a theoretical proposal which allows for applicable theory-based propositions and which is then briefly applied to the German economy over the considered time period.

The rest of the paper is organized as follows. In Section 2, we discuss the relevance of capital stock matrices for the empirical and theoretical analysis of classical price theory, and we briefly outline the sectoral composition of the German economy. In Section 3, we argue that the standard Neoricardian assumption of uniform profit and wage rates is unwarranted for the German economy 1991-1999. Then, we propose an alternative theoretical framework to derive supply-side classical price equations, which explicitly incorporate the capital stock matrix as well as profit and wage differential. In Section 4, we prove two main theoretical propositions concerning the properties of this augmented price system: first, we show that the standard distributive trade-off continues to hold in this more general context. Second, we prove that technical change shifts the augmented wage-profit curve allowing for Pareto improvements in the distribution of income. In Section 5, we provide an example of the empirical measurement of wage-profit curves for the German economy 1991-1999. Section 6 concludes.

2 Neoricardian theory and empirical analysis

In this paper, we consider the German economy as it has been investigated in detail in Kalmbach et al. (2005) on the basis of a seven sectoral input-output representation as shown in table 1.

We use their data to analyze systems of prices of production from the theoretical as well as from the empirical point of view in order to illustrate the procedures that underlie the Classical price theory by means of a concrete example, namely the German economy for the years from 1991 to 1999. We thus have 9 input-output tables (as exemplified in table 2 for the year 1995) at our disposal, which show in

[2]The introduction of capital stocks in the definition of prices of production is not novel. It figures prominently, for example, in the approach proposed by Shaikh (1998). Yet, in the latter contribution wage and profit rate differentials are not considered.

Table 1:

1 :	Agriculture	1.33
2 :	Manufacturing, the export core	12.37
3 :	Other manufacturing	22.55
4 :	Construction	6.29
5 :	Business-related services	21.36
6 :	Consumer services	23.35
7 :	Social services	12.75

Table 1: The 7-sectoral structure of the German economy (in 1995).

Note: The numbers in the last column are the sectoral output shares (in percent of real GDP) for Germany in 1995. Note that the above sectoral structure is functionally oriented and exhibits besides private also public production activities, see Franke and Kalmbach (2005) for details.

their $i-$th column the intermediate inputs of the industry i. Of course, there is also a vector of labor inputs (not shown) employed in the production activities of these seven branches of the German economy. All entries are calculated in terms of prices of a base year and measured per one million Euro of output value.[3]

Thirty two 36-sectoral input-output tables, of the qualitative form exemplified in our Table 2, and the corresponding vectors of labor inputs, from the OECD database (for nine countries from the period 1986 – 1990) served as data in Han and Schefold (2006) to investigate paradoxes of capital theory from the empirical point of view. To each input-output table the conventional definition of prices of production

$$(1+r)pA + w\ell = p$$

was applied in order to calculate the wage-profit curve $r(w)$ which can be associated with such input-output structures. The thirty-two given 36-sectoral data sets are compared by Han and Schefold in pairs, so for example in their Table 1 they compare Germany 1990 and Canada 1990. This gives, when they are mixed together, two processes for each industry and thus 2^{36} possible economy-wide techniques, or mixed input-output matrices, on the basis of which the envelopes of the above wage-profit curves can be calculated. Thus there are $\binom{32}{2}$ pairs of input-output matrices considered in the paper by Han and Schefold (2006). The main purpose of this paper is not to provide a detailed critical analysis of their circulating capital approach to

[3]See Franke and Kalmbach (2005) for the details.

Table 2:

	1	2	3	4	5	6	7
1 :	0.028	0.000	0.045	0.000	0.000	0.002	0.002
2 :	0.090	0.282	0.050	0.022	0.003	0.008	0.011
3 :	0.142	0.232	0.324	0.287	0.030	0.055	0.065
4 :	0.007	0.003	0.006	0.017	0.006	0.028	0.016
5 :	0.142	0.121	0.140	0.107	0.332	0.134	0.096
6 :	0.036	0.053	0.051	0.108	0.072	0.152	0.049
7 :	0.031	0.006	0.011	0.007	0.007	0.013	0.024

Table 2: Technological coefficients of the 7-sectoral aggregation (Germany, year 1995).

the empirical investigation of wage-profit frontiers. It is however important, from the methodological viewpoint, to note that such comparisons of mixed intermediate input-output matrices do not make sense from the empirical perspective. This is not because the rates of profit they calculate on this basis are much too high, but because of the neglect of the capital tied up (or the capital advanced) behind their intermediate input columns. We do not use the term fixed capital here, but follow Bródy's (1970) Leontief accounting distinction between 'capital consumed' and 'capital advanced', and the related turnover times, since the latter term also applies to intermediate inputs and is not as physically oriented as the term 'fixed capital' (often simply viewed as machinery) suggests. Moreover, the factual treatment of stocks of capital items is not a special case of the factual treatment of joint production as suggested by Sraffa (1960) or von Neumann (1945), see Bródy's (1970) stock-flow considerations and related input-output literature for details.

Table 3 shows the capital stock matrix $K = (k_{ij})$ for the German economy[4] corresponding to the matrix of intermediate inputs shown in table 2.[5] The single entries k_{ij} of the matrix K represent the amount of capital produced by sector i, where i can represent machinery, buildings, and also inventories of so-called circulating capital goods (raw materials), that is tied up (or advanced) in the production of sector j (per 1 million Euro output in 1995 prices). The last row in the table com-

[4]Note here that the housing stock (of consumers) is not included in the capital – output ratios shown in the last row which explains the relatively low values of these ratios.

[5]For details of its derivation, see Flaschel (2008, ch.8). These details do not matter here, however, because our objection to the approach proposed by Han and Schefold (2006) is primarily a methodological one.

Table 3:

	1	2	3	4	5	6	7
1 :	0.0156	0.0043	0.0055	0.0021	0.0080	0.0111	0.0086
2 :	0.3705	0.1028	0.1312	0.0493	0.1897	0.2625	0.2044
3 :	0.4959	0.1377	0.1756	0.0660	0.2539	0.3513	0.2736
4 :	0.9031	0.2507	0.3199	0.1202	0.4624	0.6398	0.4982
5 :	0.2217	0.0615	0.0785	0.0295	0.1135	0.1570	0.1223
6 :	0.0835	0.0232	0.0296	0.0111	0.0427	0.0591	0.0460
7 :	0.0074	0.0020	0.0026	0.0010	0.0038	0.0052	0.0041
$\sum$:	2.0975	0.5823	0.7429	0.2792	1.0739	1.4861	1.1572

Table 3: Capital stock matrix K (Germany, 1995).

putes the column sums. The last number in column 2, for example, indicates that the manufacturing (export core) sector has capital goods installed that are worth 0.5823 million Euro in 1995 prices (per 1 million Euro output in 1995 prices). Thus, the column sums can be said to represent the sectoral capital-output ratios (the ratios of capital to gross output, more precisely). What kind of economies do the mixed intermediate input techniques used in Han and Schefold (2006; see, for example, table 1) represent if the capital that is tied up in the background of their current application is in Canada in the case of a German process, and vice versa? But even in the case of our nine years of input-output data for a single economy (Germany, 1991 – 1999), what is the meaning of combining the intermediate inputs of the seven sectors from different years into one single technique, and its hypothetical wage profit curve, without recognizing that these inputs may be specific to the capital that is tied up in their production process? Our basic conclusion here is that the compositions of production processes of the 36 sectors considered in Han and Schefold (2006) are devoid of any economic content and that different ways of considering factual wage-profit curves, and the prices of production they give rise to, must be found in order to investigate the German economy for the period 1991 - 1999 as briefly described above. This task is dealt with in the following section.

3 Theoretical aspects of wage-profit curve measurements

Let w_i denote the nominal wage paid in sector $i, i = 1, ..., 7$, and let w_m denote the nominal wage in an arbitrarily chosen reference sector, $m \in \{1, ..., 7\}$; and let a similar notation hold for r_i and r_m, respectively (for a given base year, say 1995). In this section, we consider prices of production on the basis of given wage and profit rate differentials - respectively, w_i/w_m and r_i/r_m, - between the industries that make up our input-output world. The assumption of rigid wage differentials may be justified either along the lines suggested first by Keynes (1936), or just as a first approximation to a fuller analysis, which would endogenize wage differentials and consider their variations over time. The assumption of a given hierarchy of profit rates (relative to the rate r_m) is an empirically oriented substitute for the uniform rate of profit assumed by the Neoricardians for which the following table for Germany, 1991 – 1999, based on the above type of input-output data gives no justification at all.[6]

	1	2	3	4	5	6	7	average
1991 :	3.75	7.79	11.51	51.56	19.49	16.84	4.68	13.82
92 :	5.26	4.04	9.06	57.49	17.78	16.27	5.19	12.93
93 :	4.70	−0.21	7.14	51.13	17.37	15.62	4.71	11.91
94 :	5.62	1.87	7.17	52.52	17.84	16.24	5.67	12.72
95 :	5.97	5.60	5.51	47.39	16.85	17.32	5.23	12.81
96 :	7.05	4.28	6.56	42.56	16.46	17.97	5.20	13.05
97 :	7.44	6.87	7.10	40.54	16.70	18.37	4.99	13.50
98 :	6.77	7.86	7.71	39.88	17.76	18.17	5.05	13.84
99 :	5.83	5.73	8.19	40.17	18.01	17.74	4.85	13.67

Table 4: German sectoral profit rates (profits per unit of capital goods advanced) over the 1990's

From Table 4 it emerges not only that the assumption of a uniform rate of profit does not hold in the German economy in the 1990s, but - even granting the limited length of the time series - also that there is no tendency for profit rates to converge (not even partially; to a certain extent profit rates seem to co-move, actually). We admit that capital stock calculations are tedious and may be fairly inaccurate. There may be a tendency towards the leasing of capital equipments which are therefore indeed tied up in another sector. The construction sector may also exhibit a large amount of semi-finished products which should be looked at in detail. Such details are however secondary here for the conclusion that the assumption of a uniform rate of profit across these fundamentally different sectors is not of much help in the investigation of the evolution of the German economy. In general, the differentials in profit rates emerging from Table 4 are of such a significant magnitude that they seem unlikely

[6]Note the fact that the assumption of fixed wage and profit differentials is - mathematically speaking - a generalization of the Sraffian model which also assumes fixed 'differentials', with all ratios being equal to one. Actually, at this point, we might even invoke the 'argument of sufficient rigor in comparison' proposed by Han and Schefold (2006, p.749), according to which, although empirical analysis must be as rigorous as possible, it need not be more rigorous than the alternative approach that is criticised, the Sraffian approach in our case.

to be the product of measurement errors only. Furthermore, it is worth noting that similar empirical findings are obtained by a number of other authors (see Shaikh, 2009, and references therein) for various other countries, too.

The assumption of uniform profit rates is likely to become less and less convincing the higher the degree of disaggregation, in particular when it comes to the separation of coexisting modern, average and outdated processes of production. The following construction attempts to avoid such a counterfactual assumption and the wage-profit curves that are derived from it.

We now choose with respect to table 4 a base year (1995) and a reference sector m, and fix the then prevailing empirical ratios $q_i^r = r_i/r_m$ and $q_i^w = w_i/w_m$. This is by no means innocent, since some of the proportions do vary considerably from one year to another, but it seems reasonable as a first approximation. On this basis, we define the time-independent diagonal matrices

$$Q_m^r \;=\; \mathrm{diag}\,[q_1^r, q_2^r, \,\dots\, , q_n^r]\;, \qquad Q_m^w \;=\; \mathrm{diag}\,[q_1^w, q_2^w, \,\dots\, , q_n^w]$$

Let A^δ denote the depreciation matrix corresponding to the capital stock matrix K. The above definitions can be used to give rise to a system of augmented supply side price equation that differ from standard Sraffian prices of production because profit and wage differentials are taken into account, but also because capital stock and capital stock depreciation matrices are explicitly incorporated in the formulation of prices of production.[7]

$$p = p\,(A + A^\delta) \;+\; w_m\,\ell Q_m^w \;+\; r_m\,pKQ_m^r \tag{1}$$

Equation (1) employs fixed wage differentials when it varies the reference wage rate w_m, so that a rise in the reference wage rate here moves up all sectoral wage rates (and thus the general wage rate) in constant proportions. More importantly, however, the supply side prices it defines give such wage changes more influence on prices in sectors with relative high profitability compared to the conventional prices of production discussed in Section 2 above (and vice versa). The same amount of capital advanced thus has a relatively higher impact on the output price of a given sector, the higher the rate of profit in this sector. This can be looked at as if a bigger market power gave an industry more 'elbowroom' with respect to their output price.

Equation (1) can be treated similarly to the ordinary prices of production matrix equation of section,[8] but – and this is an important distinction – it has no close contact to aggregate measures of capital intensity any more, because of the distortions

[7]The ratio K_{ij}/A_{ij}^δ can be interpreted as the turnover time τ_{ij} of the capital stock item i in the industry j, where again i may represent machinery, buildings, but also the inventories that are related to so-called circulating capital goods (raw materials, etc.). See Bródy (1970) for a detailed discussion of such turnover times. The matrix A_δ is considered in more detail in Flaschel (2008).

[8]We have checked that for the base year 1995, where the nominal and real magnitudes of the input-output tables coincide, eq. (1) is indeed satisfied with $p = (1, \,\dots\, , 1)$ for the given wage rate w_m of this year.

induced by the markup structures incorporated in the matrices Q^w_m, Q^r_m. Instead, when varying their calculated rate of profit r_i, we keep sectoral profit rate (and wage) differentials constant, that is we keep something like the 'degree of monopoly' or market power, the risk structure, the markup structure constant in each sector. Variations in relative profit rates need however to be considered in a second step.

Setting $w_m = 0$, an eigen-value equation is obtained which allows us to compute sector m's maximum rate of profit R_m (given the matrix of empirical ratios Q^r_m) as the reciprocal of the dominant eigen-value $\lambda^\star$ of a suitable nonnegative matrix, i.e.:

$$R_m = 1 \,/\, \lambda^\star[\, KQ^r_m \,(I - A - A^\delta)^{-1}] \tag{2}$$

For $0 \le r_m < R_m$, prices in sector-m labor commanded are determined as

$$p^w \;=\; p^w(r_m) = \ell Q^w_m \,(I - A - A^\delta - r_m \, KQ^r_m)^{-1} \tag{3}$$

the corresponding real wage rate ω_m of sector m, measured in terms of a given consumption basket c, is given by

$$\omega_m \;=\; \omega_m(r_m) = 1 \,/\, p^w(r_m)\, c \tag{4}$$

with $\sum_i c_i = 1$, so that the unit is 1000 Euro per job per year.

We briefly indicate that the inverted matrices we have used above are indeed nonnegative and thus do give rise to the standard Leontief multiplier formula[9]

$$(I - B)^{-1} = \sum_{v=0}^{v=\infty} B^v \tag{5}$$

with respect to the nonnegative matrices they are based on. The basic assumption from which these features can be derived is the assumption of a positive price-vector such that $p - p(A + A^\delta) > 0$ holds.[10] This assumption implies that $A + A^\delta$ is nonnegatively invertible and thus a candidate for the multiplier equation (5). The matrix $KQ^r_m \,(I - A - A^\delta)^{-1}$ is therefore nonnegative and it is characterized by a vector inequality

$$p^*(A + A^\delta + r_m KQ^r_m) < p^*, \quad r_m < R_m$$

if $p^* > 0$ is the dominant eigenvector belonging to the eigenvalue λ^*. This implies that the matrix $B = A + A^\delta + r_m KQ^r_m$ is also nonnegatively invertible.

This section has proposed a concept of the wage-profit curve, as formally defined by equations (2) and (3), and corresponding 'normal' prices with frozen relative positions between the workers and between the capitalists of the seven sectors considered. We suggest that such an approach (based on capital consumed and capital

[9] The details of the theory of nonnegative matrices are provided in Nikaido (1968).

[10] For all vectors $x = (x_1, \ldots, x_n)$ and $y = (y_1, \ldots, y_n) \in \mathbb{R}^n$, $x > y$ if and only if $x_i > y_i$ $(i = 1, \ldots, n)$. Since this paper is methodological in orientation we do not go into questions of semipositivity here, but assume that everything is expressible with strict vector inequalities.

advanced expressions) can be of empirical relevance when for example the consequences of a change in the base wage w_m are to be investigated. An example for such an application of our revised supply-side pricing scheme and its implications for income distribution between capital and labor is investigated in the next section.

4 Some basic propositions

In this section we discuss some basic features of our new definition of 'prices of production' and of the wage-profit curve they imply. These propositions concern, first of all, some implications of variations in income distribution in a given technological environment and then consider some implications of technical change at the 'modern margin' of the economy which may not easily be detected through the use of input-output tables (including labor inputs, depreciation matrices, and capital stock matrices) as they are currently measured.

Proposition 1 provides some insights on price changes and income distribution trade-offs, of empirical importance primarily around the actual position of the economy, and does this for the actual set of production processes of a certain year (with their different age structure of the capital advancements that represent 'fixed capital items' and which underlie the averages reported in the year's input-output data).

Proposition 1:

1. *The price vector $p^w(r_m)$ is strictly increasing with the reference rate of profit r_m.*
2. *The real wage rate $\omega_m(r_m)$ is a strictly decreasing function of the reference rate of profit r_m.*

Proof: Part 1. The result directly follows from the Leontief multiplier formula

$$(I - B)^{-1} = \sum_{v=0}^{v=\infty} B^v$$

applied to the matrix $B = A + A^\delta + r_m KQ^r$, since the latter gives:

$$p^w = \ell Q_m^w \sum_{v=0}^{v=\infty} (A + A^\delta + r_m KQ_m^r)^v.$$

Part 2. The result follows directly from part 1 and equation (4). ■

We now consider the actual situation of the base year 1995. By definition, the actual prices vector $p(= e)$ in 1995 fulfills the vector equation

$$p = p(A + A^\delta) + w_m \ell Q_m^w + r_m pKQ_m^r$$

Our next task is to investigate the effect of technical change on the wage-profit curve and the structure of distributive conflict. Let us assume that new techniques (indexed by an asterisk $*$) are entering one, some, or even all sectors of the economy such that the following inequality holds

$$p_j > p(A^* + A_j^{\delta *}) + w_m \ell_j^*(w_j/w_m) + r_m p K_j^*(r_j/r_m)$$

for all sectors j where innovation occurs. The profit rate of the new entrant is therefore always larger than the average profit rate of the firms already coexisting in this sector (the capital advancements of which are not yet fully depreciated).

Proposition 2:

Let ω^* and p^{w*} denote, respectively, the real wage rate and the price vector of the production price system (3) where the averaging columns of the matrices $\left(A, A^\delta, \ell, K\right)$, are replaced by the columns $\left(A^*, A^{\delta *}, \ell^*, K^*\right)$ of the new production processes. We then have:

$$\omega_m^*(r_m) > \omega_m(r_m) \quad [p^{w*} < p^w]. \tag{6}$$

Proof: The inequalities

$$p_j > p(A^* + A_j^{\delta *}) + w_m \ell_j^*(w_j/w_m) + r_m p K_j^*(r_j/r_m)$$

can be transformed into equalities again (with the actually given prices p)

$$p_j = p(A^* + A_j^{\delta *}) + w_m \ell_j^*(w_j/w_m) + r_m p \tilde{K}_j^*(r_j/r_m)$$

by increasing one (some) entries in the columns K_j^* towards a new vector $\tilde{K}_j^*$ in an appropriate way. This gives rise to the vector equation

$$p = p(A^* + A^{\delta *}) + w_m \ell^* Q_m^w + r_m p \tilde{K}^* Q_m^r$$

where the innovating sectors exhibit the new production processes only. Rearranging terms this gives rise to an equation as in (3)

$$p^w = p^w(r_m) = \ell Q_m^w (I - A^* - A^{\delta *} - r_m \tilde{K}^* Q_m^r)^{-1}$$

and thus implies

$$\begin{aligned} p^w &= \ell Q_m^w \sum_{v=0}^{v=\infty} (A^* + A^{\delta *} + r_m \tilde{K}^* Q_m^r)^v \\ &> \ell Q_m^w \sum_{v=0}^{v=\infty} (A^* + A^{\delta *} + r_m K^* Q_m^r)^v \\ &= p^{w*} \end{aligned}$$

Inserting this into equation (4) proves the proposition. ■

In other words, Proposition 2 states that the immediate replacement of all old processes by the considered innovations allows for Pareto improvements in the distribution of income between capital and labor (and for falling prices of production after all old techniques in the changing sectors have disappeared). Note that the new techniques are added in the actual calculation of the implied averaging input-output tables to the ones that are not yet fully depreciated, i.e., the wage-profit curve of this input-output table is not the one that is given by $\omega_m^*(r_m)$ due to the neglect of the old techniques (of the innovating sectors) in the calculation of $\omega_m^*(r_m)$. This latter wage-profit curve can only be obtained from standard input-output tables when the old techniques of the innovating sectors have become extinct. We consequently get that the wage-profit curves of actually observed input-output systems are hybrid in nature (are mixing old and new technology in some averaging way). They therefore cannot be easily utilized to consider questions of reswitching and the like, since technical change is only working on the margin of their input-output data so to speak. It is also not at all obvious here whether the proven outward shift in the curve $\omega_m^*(r_m)$ is correctly mirrored by the wage-profit curve (4) of the implied input-output tables (due to their hybrid character).

5 Wage-profit curve measurement: An example

In this section, we apply the above wage-profit curve definition (4) to the German economy over the years 1991 – 1999 as characterized by the input-output structures discussed in the preceding sections. We have in principle 9 such curves at our disposal (since we do not mix processes from different years into a single input – output structure), but in figure 4 we only show the results for four selected years in order to get a clearly visible impression of their differences. The results we obtain in this way are astonishingly straightforward and can be generalized to the wage-profit curves not shown in Figure 1.

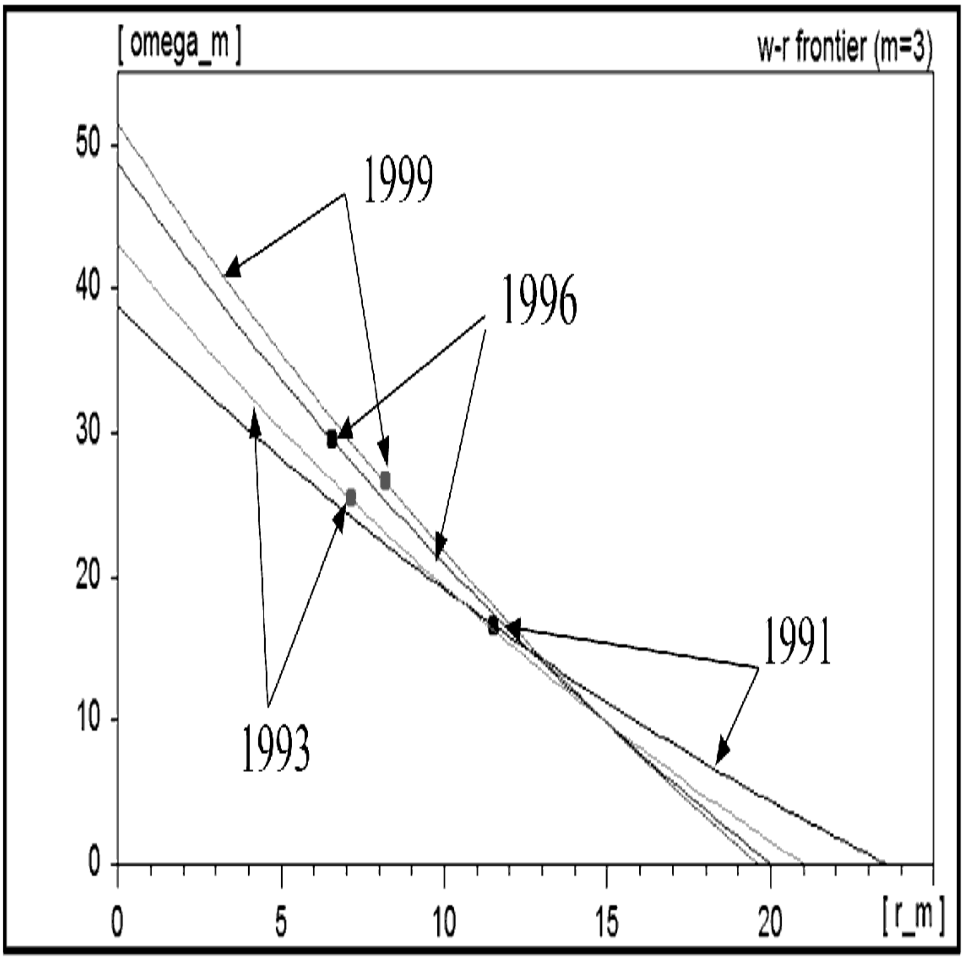

Figure 1: Four wage-profit curves and their envelope

First of all, figure 1 shows that wage-profit curves are fairly close to straight lines, independently of the special choice of scaling, a result already stressed by Shaikh in many contributions, see for example Shaikh and Tonak (1994), Shaikh (1998). Secondly, the order of techniques is such that – in pairwise comparison, not always on their envelope – a sufficiently declining wage ω_m would lead us back in time from 1999 to 1996 to 1993 to 1991. Note moreover that the actual positions of the economy in these 4 years are marked by a filled square on these curves. To a certain degree, they reflect the business cycle of the period under consideration and therefore need not be ordered in a monotonic fashion as far as profit rates are concerned (they are however by and large monotonically increasing in their wage rates as time goes by). But the really important result here is that a decline in the wage rate ω_m would not necessarily imply what neoclassical theory claims to happen in the context of a given neoclassical production function, for only the years 1991 and 1999 are positioned on the envelope of the wage-profit curves shown. This result, however, is in our context not just based on factor substitution, but also (more importantly) on the occurrence of technological change.

If a sufficiently big decrease of the wage rate did take place we would describe the underlying situation and its implications as follows. Each of the considered industries has capital tied up in its production process in form of modern, medium ranged, and old production processes. In the form of a time-to-rebuild process, we would then expect a return back to older processes so that the profile of processes that characterizes a given industry would start shifting back to older production structures or even techniques that are no longer in use. An example where this happens is the low wage income segment of actual economies where labor-using capital-saving processes can be observed. Along these lines, empirically-based calculations of wage-profit curves can be meaningful, but not in a framework where wage rates and profit rates are assumed to be uniform and are measured in their mutual interdependence on the basis of arbitrary compositions of intermediate input structures from different countries and different times.

It remains to be stressed here that the slopes of the considered wage-profit curves no longer measure something like capital intensity as in the Neoricardian approach, because they are distorted now by the varying markup structures incorporated in the matrices Q^w_m, Q^r_m (due to a number of reasons such as varying degrees of monopoly, differences in risk structures, differences in the relationship to new markets, and more). We conclude that a Leontief-Bródy approach (augmented by profit rate differentials) is better for applied analysis than the purely theoretical orientation that characterizes the Sraffa / von Neumann world.

6 Conclusions

Han and Schefold (2006) consider actual 36-sectoral intermediate input-output tables for a variety of countries and years (32 tables altogether) which they interpret as sets of blueprints from which 2^{36} possible economy-wide hypothetical techniques or mixed input-output matrices can be formed from the mathematical perspective. The 2^{36} wage-profit curves obtained from such virtual combinations of intermediate input tables (and labor inputs) are however not relevant for the decision making of firms as far as the choice of single activities are concerned. On the one hand, the information incorporated in these curves is not available to firms due to the economy-wide foundation these virtual curves exhibit by definition (which cannot be accessed from a partial perspective). On the other hand, the costs of intermediate inputs are only one element in the investment decision of firms which may be primarily concerned with the search for labor-saving (possibly capital-using) long-lasting capital goods where possibly the accompanying change in used up raw materials is of a marginal nature. In our view, the calculation of such virtual intermediate-input-based wage-profit curves is therefore not helpful in the investigation of actual sectoral technical change and its economy-wide consequences.

Our alternative proposal for the measurement of empirically relevant wage-profit curves (and the 'prices of production' that accompany them) was based not only on the addition of capital stock matrices K and capital stock depreciation matrices A^δ, but also on the recognition of the fact that there is no tendency towards uniform wage rates and – more importantly – uniform profit rates, at least for the German economy between the years 1991 and 1999. The assumption of such uniformities significantly simplifies the calculation of prices of production and the wage-profit curves associated with them, but since these assumptions are utterly unrealistic, as was already argued in Farjoun and Machover (1983), they are not of help in applied theory.

One may however argue with respect to our alternative proposal of considering input-output structures of the form A, ℓ, K, A^{δ}[11] in place of only A, ℓ that this also provides a set of blueprints from which firms may choose new input-output combinations from their sectoral (partial) perspective, implying that the analytical procedure of Han and Schefold (2006) can still be applied. In our view, however, such an argument is not really compelling.

First, there is the problem of the depreciation of the old production activities. Such processes may be slow and may hide the role of the most modern techniques in the data supplied by input-output statisticians. Even if our seven-commodity world was taken literally, they would imply that the old and the newly chosen production activities coexist for some time, so that the reported input-output structures are necessarily averages over coexisting vintages of techniques (even if all aggregation problems within highly aggregated input-output tables are ignored). Secondly, investment criteria can be – in the presence of 'fixed capital' – quite complex[12] and in any case are not based on the comparison of economy-wide averaging wage-profit curves. They should have the implication of providing more room for income distribution for capital and labor through technical change, but this implication cannot be used as a criterion for the choice of technique. Finally, wage-profit curves can only be defined in a meaningful way if actual wage and profit rate differentials are taken into account and thus they are by no means purely technologically-based constructs.

The wage-profit curves we have defined in this paper are ex post constructs which show the implications of changes in the chosen baseline real wage for a certain scheme of production prices (exhibiting a given set of wage and profit rate differentials) and also for the wage-profit curve associated with this set. They show this for a given technology of a given economy in a given year. On this basis it may then be discussed (as done in Proposition 2) how technical change that currently occurs modifies the scope within which the conflict between capital and labor over income distribution takes place. Our figure 1 demonstrates in this regard that the realized tuples of income distribution of subsequent years have been always outside the reach of the wage-profit curve that were preceding this situation.

[11]Ignoring here the matrices Q^w_m, Q^r_m for the time being.
[12]We have formulated an averaging example in Proposition 2.

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