

ARBITRAGE SEQUENCES AND LEIJONHUFVUD 'S CORRIDOR

HYPOTHESIS

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ABSTRACT: In the Leijonhufvud synthesis, economies are self-adjusting within some ‘corridor’, but not outside. In a previous paper, Cross et. al. (2010), used combinatorial analysis to see if arbitrage sequences involve a smooth convergence onto an equilibrium in which the law of one price holds. They found that arbitrage sequences tend to be periodic in nature. In the present paper we argue that this result is corrosive for the idea that economies are self-adjusting within the ‘corridor’ postulated by Leijonhufvud .

KEYWORDS: Arbitrage sequences, stability, equilibrium, periodicity, path-dependency, Leijonhufvud .

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JEL CLASSIFICATION:

In 1934 Keynes chose the sub-title ‘Is the Economic System Self-Adjusting?’ for his contribution to a series of radio talks on the pressing issue of the day, ‘Poverty in Plenty’ (Keynes, 1934). Whilst there are different interpretations as to exactly why economic systems are not, or may not be, self-adjusting, the at least potential absence of self-adjustment to a ‘full employment’ equilibrium is something the different branches of Keynesian economics have in common. In this paper we discuss how arbitrage fails to ensure a smooth convergence to the equilibrium described in the law of one price. Following the results obtained in the combinatorial analysis of arbitrage sequences in Cross, Kozyakin, O’Callaghan, Pokrovskii and Pokrovskiy (2010), we argue that the lack of self-adjustment is endemic to economic systems, occurring within the ‘corridor’ postulated by Leijonhufvud as well as outside it.

The paper is organized as following. In **Section I** of this paper we discuss the role played by arbitrage in different varieties of Keynesian economics, making use of the ‘hydraulic’, ‘fundamentalist’ and ‘reconstituted reductionist’ classifications of Coddington (1976). The contrast is between arbitrage failure being in the background in the ‘hydraulic’ and ‘fundamentalist’ traditions, but in the foreground in ‘reconstituted reductionism’. In **Section II** we consider whether arbitrage processes are likely to lead to smooth convergence onto an equilibrium in which the law of one price holds. Following our earlier combinatorial analysis of arbitrage sequences (Cross et. al., 2010) we find that such sequences tend to be periodic in nature, providing endogenous reasons as to why there is not convergence onto a law-of-one-price equilibrium. **Section III** considers the implications of this finding for

Leijonhufvud's 'corridor' hypothesis variant of 'reconstituted reductionism'. In line with the previous critical assessment of De Antoni (2006), we find that self-adjustment problems are pervasive within, as well as outside, the 'corridor'.

Section I: First Principles in Keynesian Economics

For the purposes of this paper the Coddington (1976) depiction of three traditions, that attempt to encapsulate what was distinctive in Keynes, provides a useful point of departure. "The matter may be expressed cryptically in terms of Keynes' "long struggle to escape". We may say that what he escaped from was (unreconstituted) reductionism; what he escaped to was the hydraulic approach; and what he went through in the process of struggle has been preserved in the fundamentalist approach" (Coddington 1976. p. 1272).

The 'hydraulic' tradition, initially captured in the income-expenditure and IS-LM models, pictures the economy in terms of disembodied stocks and flows. The Phillips MONIAC machine provides the most vivid exemplification of the hydraulic nature of this tradition (see the papers in Leeson, 2000). The 'psychological propensities' driving private sector investment and consumption can lead aggregate demand to fall short of the level required to elicit 'full employment'. Prices are taken to be fixed, or at least sticky, so there is little by way of explicit analysis of arbitrage processes in this framework.

In the 'fundamentalist' tradition the key problem is taken to be the intractable nature of the uncertainty, regarding the unknowable future, that surrounds the

decision taking by economic agents. Keynes' own response to reviewers of his *General Theory* stressed the endemic nature of such intractable uncertainty (Keynes, 1937). Arbitrage failure can play an important role in this interpretation of how the fragile expectations, driven by emotions as well as reason, which drive economic decision-taking lead to non-equilibrium outcomes, but there is little by way of explicit analysis of arbitrage sequences.

It is in the third 'reconstituted reductionist' tradition that arbitrage failure occupies the centre of the stage. Building on the analysis of the effective demand failures that can arise from producers (Patinkin, 1965) and consumers (Clower, 1965) being quantity-constrained when trading at disequilibrium prices, in Leijonhufvud (1968) arbitrage failure, in the form of the relative prices of producer goods being 'wrong' in relation to those for consumer goods, plays the key role in the explanation of why economic systems may not be self-adjusting to a 'full employment' equilibrium. This approach spawned the analysis of disequilibrium trading to be found in such as Barro and Grossman (1976) and Malinvaud (1977).

Leijonhufvud (1973, 2009b) subsequently produced a 'corridor hypothesis' variant of reconstituted reductionism. This hypothesis proposed that self-adjustment prevails within the bounds of some 'corridor', but not when an economic system is perturbed sufficiently to find itself outside the 'corridor'. "Within some 'corridor' around an equilibrium time-path, the usual adaptive market mechanisms would operate to co-ordinate activities. But further away from equilibrium, effective demand failures would impair the system's ability to restore itself to a co-ordinated state, and beyond the bounds of the corridor it would languish in far-from-equilibrium states

indefinitely unless salvaged by effective policy interventions. As you might surmise, this corridor hypothesis was heartily disliked by Keynesian and free-market fundamentalists alike. It is just unattractive to people with an ideological bent.” (Leijonhufvud 2009a, p. 3).

So the Leijonhufvud corridor hypothesis can be considered as a ‘Leijonhufvud synthesis’. Within the corridor self-adjustment works, and the world is very much as described in the negative feedback world found in the neoclassical models of ‘full employment’ equilibria. Outside the corridor self-adjustment does not work, and the world is as described in Keynesian models where positive feedback involves the amplification of effective demand failures, generating far-from-equilibrium states.

Section II: Arbitrage Sequences and the Stability of Equilibria

In an Arrow-Debreu equilibrium the law of one price holds. Even if one is satisfied with the restrictive conditions required to yield existence proofs, such equilibria would be pretty irrelevant if stability proofs could not be provided. Economic systems would not converge on such equilibria if they did not start off at such equilibria in the first place.

There are two main approaches to stability of general equilibrium analysis (see Fisher 1981, 1989). In one approach stability is achieved by the good offices of an unpaid auctioneer who calls a set of relative prices in order for agents to reveal their demands and supplies. Through a Walrasian *tâtonnement* process the relative price vector is then adjusted until excess demand equals zero on all markets. No trade is

allowed to take place at disequilibrium prices, so there are no arbitrage opportunities for agents to exploit. The law of one price result holds because of the no-trade-at-disequilibrium-prices feature imposed on the *tâtonnement* process conducted by the *deus ex machina* auctioneer.

In the other main approach, stability is achieved by the arbitrage operations of economic agents. A set of ‘wrong’ relative prices, in which the law of one price does not hold, would involve the existence of profitable arbitrage opportunities. The key question here is ‘can one expect to prove that an economy with rational agents conscious of disequilibrium and taking advantage of arbitrage opportunities is driven (asymptotically) to any equilibrium, Walrasian or constrained?’ (Fisher 1989, p. 86-87). Fisher uses the restriction of ‘no favourable surprise’ as a means of demonstrating that a cessation of exogenous surprises or shocks could lead to convergence to equilibrium. For this stability result to hold it has to be shown that the endogenous processes associated with the exploitation of arbitrage opportunities involve convergence to equilibrium. A neglected issue has been the analysis of the arbitrage sequences or chains that could be involved if such arbitrage-driven convergence to equilibrium were to take place. It is to that issue that we now turn.

In what follows we provide a non-technical summary of the main results of the combinatorial analysis of arbitrage sequences provided in Cross et. al. (2010). This analysis considers the case of a foreign exchange (FX) market in which currency traders/arbitrageurs initially know only the exchange rates for their own domestic currency. Thus, in a three currency example, the dollar trader would initially know the exchange rate for the dollar (D) against the euro (E) and the pound sterling (S), but

not the exchange rate for the euro against sterling. The three principal exchange rates are r_{DE} , r_{DS} and r_{ES} , where r_{DE} is the number of euros that can be bought with one dollar, and so on. The reciprocal exchange rates are $r_{ED}=1/r_{DE}$, and so on. The assumption that traders/arbitrageurs have limited initial information on arbitrage opportunities is not implausible. Currencies, goods and assets are not traded on a single exchange. There are various trading posts such as commodity and stock exchanges. Other trades, including many foreign exchange deals, are conducted ‘over the counter’ in direct transactions that bypass formal exchanges. As a result of this segmentation, arbitrage operations tend to involve specific arbitrage networks: ‘various clienteles trade on different exchanges, and very few clients trade on more than one exchange, let alone on all of them simultaneously’ (Ratic and Zigrand, 2008, p.3). FX dealing rooms tend to concentrate on trades involving the domestic currency, so the assumption that FX traders/arbitrageurs initially are not aware of the cross exchange rates between other currencies does not seem to be too far from the mark of reality.

In the three-currency case, arbitrage operations are straightforward, a single arbitrage trade being sufficient to bring the FX market back into an equilibrium of balanced exchange rates in which the law of one price holds. Consider the case where a dollar trader could make a profit by exchanging a dollar for r_{DS} units of sterling, and then exchanging the sterling for r_{SE} euros where the ‘r’ stands for exchange rate. Thus the following inequality holds:

$$r_{DS}.r_{SE}>r_{DE} \quad (1)$$

The order in which the FX traders become aware of arbitrage opportunities determines which trader will take advantage of the opportunity. In the case of the

dollar trader being the first to have this information, this dollar trader would contract with the euro trader to increase r_{DE} to a new rate:

$$\mathbf{r_{DE} = r_{DS} \cdot r_{SE} = r_{DS}/r_{ES}} \quad (2)$$

and the FX market would achieve the law of one price balanced exchanged rate ensemble of:

$$\mathbf{r_{DS}/r_{ES}; r_{DS}; r_{ES}} \quad (3)$$

In the case of the euro and sterling traders respectively, being the first to recognise the arbitrage opportunity, the balanced set of exchange rates after the arbitrage would be, respectively:

$$\mathbf{r_{DE}; r_{DS}; r_{DS}/r_{DE}} \quad (4)$$

and

$$\mathbf{r_{DE}; r_{DE} \cdot r_{ES}; r_{ES}} \quad (5)$$

Once we move to a larger number of currencies, arbitrage operations are no longer so straightforward. If there are four currencies the number of principal exchange rates is $C_4^2 = \frac{4!}{2!(4-2)!} = 6$ where C stands for combinations. The number of

possible arbitrage sequences is now the number of permutations (P) of three

currencies which could be chosen from the four available, $P_{4,3} = \frac{4!}{(4-3)!} = 24$. Let the

fourth currency be the yen (Y). So the ensemble of principal exchange rates is:

$$\mathbf{r_{DE}; r_{DS}; r_{DY}; r_{ES}; r_{EY}; r_{SY}} \quad (6)$$

This ensemble of exchange rates would be balanced, and the law of one price would hold, if and only if:

$$\mathbf{r_{ES} = r_{DS}/r_{DE}; r_{EY} = r_{DY}/r_{DE}; \text{ and } r_{SY} = r_{DY}/r_{DS}} \quad (7)$$

So, for example, one of the possible twenty-four arbitrage sequences would be yen-sterling-euro, or aYSE, where the ‘a’ stands for arbitrage sequence. This arbitrage sequence would be activated, in the sense of yielding profit, when $r_{EY} < r_{SY} \cdot r_{ES}$. If the foreign exchange rates moved to $r_{SY} = r_{EY}/r_{ES}$ this arbitrage sequence would no longer be profitable.

The question is then one of whether the arbitrage sequences that can be pursued in this four-currency world involve smooth convergence to a balanced exchange rate ensemble in which the law of one price holds. This problem can be approached using combinatorial analysis and desynchronised systems theory (see Cross et. al. 2010, for the mathematical details). Given that the FX traders/arbitrageurs are initially aware only of the exchange rates for their domestic currencies, the order in which arbitrage opportunities are revealed, in the form of cross exchange rate discrepancies involving other currencies, dictates which arbitrage sequences will be pursued first. The key finding in Cross et. al (2010) is that the arbitrage sequences pursued, in the sense that they are active, tend to be periodic in nature. Thus, in contrast to the three-currency case where only one arbitrage operation is required to achieve a balanced exchange rate ensemble, the emergence of an arbitrage opportunity in the four-currency world is followed by periodicity in the active arbitrage opportunities. This means that once an unbalanced exchange rate ensemble emerges there is no smooth convergence to a balanced exchange rate ensemble where the law of one price holds.

The above finding is, as far as we are aware, novel and arises once combinatorial methods are applied to the problem of analysing arbitrage sequences in

economic systems. This periodicity result arises once we move from a three to a four currency world. The Financial Times provides daily quotes for the exchange rates in fifty-two currencies. In this real world there are $C_{52}^2 = \frac{52!}{2!(52-2)!} = \mathbf{1,326}$ principal exchange rates. The number of possible arbitrage sequences involving three currencies is given by $P_{52,3} = \frac{52!}{(52-3)!} = \mathbf{132,600}$.

The dimensions of the arbitrage problem obviously expand to huge numbers once goods and assets are considered along with currencies. We are reasonably confident that the periodicity result will also obtain once the arbitrage sequence analysis is extended to a world in which there are very large numbers of goods and assets to be considered as well as currencies.

In relation to the arbitrage opportunities involving goods, the early literature on the law of one price is coeval with that on the purchasing power parity explanation of foreign exchange rates. Empirical tests suggest that arbitrage operations exert a strong influence on exchange rates only when the price index deviations exceed about 25% (Engel, 1999). A study of the prices charged for identical products in IKEA stores in twenty-five countries revealed typical price divergences of 20-50%, differences that could not be attributed to just country or location-specific factors (Haskel and Wolf, 2001). This empirical evidence suggests that deviations from the law of one price in goods markets are more the rule than the exception.

In relation to assets, the early application of the law of one price was also to exchange rates, in the form of the interest rate parity theory of forward exchange rates, whereby the ratio of the forward to spot exchange rate between two currencies is equal to the ratio of the interest rates in the two currencies over the forward period

in question. The absence of arbitrage opportunities is seen as the cornerstone of mainstream theories in finance, being applied in the Modigliani-Miller theorem of corporate capital structure, the Black-Scholes model of option pricing and the arbitrage pricing model of asset prices (Ross, 1978). Notable deviations from the law of one price have been documented, examples being found in the comparable circumstances applying to closed-end funds, American Depository Receipts, twin shares, dual share classes and corporate spin-offs (Lamont and Thaler, 2003). Again, deviations from the law of one price seem to be as much a regular as exceptional feature of asset markets.

SECTION III: Arbitrage Failure and the Leijonhufvud Corridor

In Leijonhufvud's version of 'reconstituted reductionism', co-ordination failures, mediated by information, learning, and information problems, can result in far-from-equilibrium outcomes. A key role can be played by trade taking place at a set of 'wrong' relative prices of capital goods in relation to those for consumption goods. In this world the adjustment of quantities involves positive feedback, the less-than-equilibrium quantities traded leading to secondary effects on effective demand that amplify the deviations from equilibrium. In his 'corridor' hypothesis, however, Leijonhufvud postulates that such effects only come into play when economies are perturbed sufficiently to find themselves outside some 'corridor', inside which general equilibrium analysis prevails.

This 'Leijonhufvud synthesis' has been criticised for retaining a general equilibrium framework for analysing what happens within the 'corridor'. As De

Antoni (2006, p. 86) puts it: “Using the general equilibrium model as a benchmark, Leijonhufvud exposes himself to the risk of considering as normal what is actually artificial (the tautological neutrality of money which in his benchmark is only an artifice) and as pathological what is actually normal (the real effects of money, involuntary unemployment, the endogenous components of the cycle, and so on).”

The results described in the previous section of the present paper, principally that arbitrage sequences tend to be periodic in nature, apply within Leijonhufvud’s ‘corridor’, as well as in the ‘far-from-equilibrium’ zones outside. If our results hold, the implication is that arbitrage failure is endemic in economic systems, rather than just constituting a pathological case that can be considered as ‘Keynesian’, occurring only outside some ‘corridor’ in which general equilibrium holds sway. This means the ‘reconstituted reductionist’ approach in Keynesian economics has more general validity than Leijonhufvud’s ‘corridor’ hypothesis would suggest.

Once trade is no longer anchored as taking place at equilibrium prices, economic systems become path-dependent. “Even if speeds of adjustment are high enough that adjustment takes place quickly relative to time between shocks, it may not be the case that equilibrium analysis is well-founded. This is because of hysteresis – the path-dependent nature of the equilibrium reached in the adjustment processes studied. Economic change can happen out of equilibrium, and such change can affect the equilibrium reached. If comparative statics is to be useful, the adjustment process must not only be rapid and thus unimportant in terms of real time, it must also be unimportant in terms of its effects on equilibrium. In the present state of our knowledge, there is no basis for the belief that this is the case.” (Fisher 1989, p. 216).

SECTION IV: Concluding Remarks

In a previous paper, Cross et. al. (2010) used combinatorial methods to analyse arbitrage sequences in a four-currency world. They found that such sequences tend to be periodic in nature rather than involving a smooth convergence to an equilibrium in which the law of one price holds. If we rule out the case where an unpaid auctioneer provides the co-ordination required for general equilibrium stability to hold, which precludes arbitrage operations because no trade is allowed to take place at non-equilibrium prices, we are left with arbitrage as being the key process that might ensure stability. Instead we find that arbitrage operations do not lead to smooth convergence to equilibrium.

This means that co-ordination problems are pervasive rather than occurring only outside a Leijonhufvudian ‘corridor’. As with the neoclassical synthesis, and the more recent “new consensus”, the general equilibrium centre involved in the ‘Leijonhufvud synthesis’ does not hold. Instead, arbitrage-driven trades are likely to take place at non-equilibrium prices, and economic systems are path-dependent. In future work it would be useful to take account of Clower’s aphorism that “money buys goods and goods buy money, but goods do not buy goods” (Clower, 1967, pp. 207-208). The monetary nature of exchange relationships takes on a heightened relevance once complex arbitrage sequences are considered.

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