On companies' microeconomic behaviour:
profit rate versus economic profit

Louis de MESNARD
University of Burgundy and CNRS
(Laboratoire d’Economie et de Gestion, UMR 5118)

Address:
PEG, University of Burgundy, 2 Bd Gabriel, F-21000 Dijon, FRANCE. Email: louis.de-mesnard@u-bourgogne.fr.

Abstract.
Profit-rate maximization leads to use fewer factors —including labour— even if profits are high and it corresponds to shareholders’ financial behaviour, by contrast to economic-profit maximization which corresponds to shareholders’ strategic behaviour. This is shown in two steps.

In part 1, two types of firms are considered: those which maximize their net profit, as assumed classically in the microeconomic theory, and those which maximize their profit rate. We compare the behaviour of both types of firms by respect to output and price. If the firm is producing, the output (and the input consumption) of a profit-rate-maximizing firm is lower than (or equal to) those of a pure-profit-maximizing firm; the price of output evolves in the opposite way. The demonstration is valid for monopoly (higher price, lower input) and for perfect competition (lower input); in perfect competition with fixed coefficient of capital, the output price loses any role in the equilibrium what implies no coordination. It is also applied to the case where the capital is the total capital engaged (EVA versus ROCE) or where it is the equity (EVA versus ROE) as in part 2.

Part 2 explores how shareholders’ behaviour may influence companies’ objective. Two main cases are examined (leaving aside the questions of corporate governance or agency theory). (i) The “strategic behaviour”. Shareholders try to maintain fixed their control rate on firms: they maximize their own net income which includes companies' distributed profit. Hence companies maximize their economic profit. (ii) The “financial behaviour”. Shareholders control the composition of their portfolio, allocating freely their equity capital between firms: they maximize the return on their equity capital. Hence companies are encouraged to maximize their profit rate: they employ less factors, as labour. (iii) The “sleeping-partner” behaviour; shareholders let their equity invested in the firm for a long time, without subscribing to any new issue of shares: they maximize the return on their equity but because of their inertia, they have a small influence on the firm. The combination of these behaviours is considered. As a result, profit-sharing leads to profit-rate maximization and natural selection is in favour of profit-rate-maximizing firms.

JEL classification. L21, D21, D24, D41, D42, G11, M2.
Keywords. Behaviour, Profit, Profit rate, ROE, ROCE, Shareholder.
1. Introduction

In the neoclassical corpus, economic profit as firm’s behaviour, that is, as objective function in firm's maximization program remains widely adopted because it seems to be more natural, more intuitive and more justified. Scitovszky (1943, p. 57) says “That the entrepreneur aims at maximizing his profits is one of the most fundamental assumptions of economic theory”. In virtually all handbooks of microeconomics or industrial organization, and even in the best ones, the microeconomic firm is classically assumed to determine its output (and consequently the amount of factors consumed in the production) by maximizing its economic (or pure) profit. In his reference text book, Tirole says (1989, p. 35): “It will then be argued that even if managerial slack invalidates the profit-maximization hypothesis, the implications of this hypothesis for industrial organization need not to be erroneous”. Milgrom and Roberts (1992, p. 40) indicate that profit maximization is true except if the owner is also a customer or an input supplier of the firm, preferring better prices or conditions, or if he is one of his employees, preferring better salaries. As illustrated by this quotation of Tirole (1989, p. 35), scholars generally think that “… non-profit maximization is mainly associated with the separation of ownership and control”.

However, even if profit maximization is well established, the return on investment (or the profitability, the profit rate, etc., whatever its name is) is actually considered in the business world: in a study as old as the end of the 50’s, Lanzillotti (1958) indicates that a target return on investment may drive large companies’ long-run pricing. By respect to the return on investment, three practical paradoxes confuse the issue. (i) The performance of a firm is evaluated in terms of profitability (ratio of profit over capital engaged): the profit rate or the return on capital; but never the return on capital is assumed to be maximized: only the profit is the subject of maximization. This is true in the business world,—as underlined by Lanzillotti—but also in economic science; in the Structure-Conduct-Performance model, many authors have considered the profit rate or

1 For a discussion about this, see for example the last part of Kreps’s book (1990).
2 Even if there are exceptions as Frank’s handbook (1994, pp. 392-396): he makes the praiseworthy effort to explain (even shortly) why firms are pure profit maximizing with the following arguments: the existence of incompetent or poorly informed managers is not contradictory with profit maximization; Darwinian selection (Alchian 1950) allows some firms, which come closer to a great profit, to survive better than other which go bankrupt; money lenders want to keep their risk at minimum and prefer more profitable firms; managers receive shares of the firm to stimulate their effort.
3 The aim of this paper is not to discuss about the nature of profit. We name firm's objective the quantity maximized by firms. Slade (1994) uses the term “objective” in another sense as the function which is maximized at the market level; in the case of monopoly, both functions coincide but in any other cases, they may diverge: for example, in competition, the firms maximize the profit but the market acts as a social welfare maximizing agent; Slade calls the market objective the fictitious-objective.
4 See a discussion in annex.
5 In addition to: the stabilization of price and margins, the search of a given market share and the meeting or prevention of competition.
the rate of return as the variable to be explained by the industrial concentration (Bain 1950; Mann 1966; Schmalensee 1989).6

(ii) A famous—even now old-fashioned—regulatory rule for the monopoly consists into maximizing the profit under a constraint of “rate of return”, the “fair” rate of return, imposed by the regulator; the firm is encouraged to overcapitalize—it is the Averch-Johnson effect (1962)—, to stay below the line imposed by the regulator: the output is larger than it would be in absence of regulation. But never the natural monopoly is assumed to maximize its rate of return itself.

(iii) Above all, the recent evolution of capitalism leads an increasing number of companies to lay off their workers even when their profits have reached a record level (that is, are maximized). These companies claim that they have to adopt such a behaviour to maintain their profitability (that is, their return on capital) to the highest possible level. However, workers—who often like and even love their company—and ordinary people have some difficulties to understand why companies exhibit such a schizophrenic behaviour: for these people, the profit must be shared, or at least, a company must guarantee the level of employment when profits are so large. The examples are so numerous that they cannot be cited all but two good examples are the following, that is, are maximized. Hewlett Packard in 2005: it is a company that people really love—with a very good brand image, serious products in many branches of the market—but which have announced a huge laying off (14,500 jobs for the whole world, 10% of its manpower), while its half-yearly profits have been the highest in the company's history. Or Pfizer, one of the top pharmaceutical companies: it has decided in January 2007 to cut 10,000 jobs (again 10% of its manpower) while its profits have never been so high (19 billion dollars in 2006) and its half-yearly profits have been tripled.

The profit rate can be an objective for the firm, obviously. However, two questions come. Does choosing the profit rate as objective change anything? Can the profit rate be an objective for the firm and under what conditions this function is chosen by the firm? Our objective is not to produce an \( n+1 \)th alternative theory of profit as the object of firm’s maximization program but to show two main things.

First, we will explore the microeconomic consequences on firm’s optimal output, for monopoly and perfect competition of the switching from the pure or economic profit what corresponds, more or less, to what is assumed by the microeconomic theory to the alternative behaviour, the profit rate. It will be shown that companies which switch from the traditional economic (or pure) profit maximization to the profit-rate maximization will produce less, have a higher output price and use less factors (including labour). To show this, we use elementary microeconomics. This formal demonstration will be applied to the case where the capital is the total engaged capital and where it is the only equity, this last case being useful for the second part of the paper.

Second, even if one assumes that shareholders’ behaviour influences firm’s behaviour, an important and new result can be deduced: profit-rate maximization may come naturally as companies’ behaviour. To show this, we will focus on the global relation between all owners and all firms, letting aside all questions concerning the one-to-one relation between one shareholder and one firm, as in corporate governance or agency theory.

---

6 Alternatively, Odagiri and Yamawaki (1986) have examined the long-run movements of profit rates in Japan and the United States.

7 For a very clear exposé on the Averch-Johnson effect, read (Carlton and Perloff 1994, pp. 877-83).
(Jensen and Meckling 1976; Fama 1980; Grossman and Hart 1983)\(^8\) or of rents of other stakeholders (Milgrom and Roberts 1992; Jensen 2001).\(^9\) We will distinguish between:

- **strategic shareholders** who own a fixed proportion of firm's equity what implies that they have a strategic behaviour by maintaining their control on the firms;
- **financial shareholders** who tend to favour the management of their own portfolio, eventually moving their equity from one company to another to maximize its profitability;
- **sleeping-partner shareholders** who let stable their equity invested in the company.

It will be explained why companies may switch from one behaviour to the other. This is done in the second part of the paper.

### 2. Comparison of behaviours for pure-profit and profit-rate maximization

In the following, only the case of the monoproduction firm will be examined, with an output denoted by \(Q\).\(^{10}\) To discuss about what happens when the profit rate is maximized instead of the pure profit by respect, the demonstration will be formal, and then applied to various types of profit rates. At this step, what the capital is will not be stated: it could be the capital engaged or the equity. One assumes that \(K\) is a function of the output \(Q\), denoted \(K(Q)\) with the condition \(K(Q) > 0\) for all \(Q > 0\).\(^{11}\) \(K(Q)\) serves to buy the net assets used to produce a commodity: the nature of this function will be discussed later. \(K(Q)\) is measured either at its book (or historic or accounting) value or at its replacing value or \(ex\ \text{ante}\) when producing the output \(Q\) is decided; it is not the current market value.\(^{12}\) \(\Pi(Q)\) stands for the profit; we neglect the taxes and the exceptional charges, etc. Again, at this step, it not necessary to state what exactly this profit is but \(\Pi(Q) = R(Q) - C(Q)\) where \(R(Q)\) is the revenue and \(C(Q)\) is the cost that will be specified later, depending on what the capital is. If \(\pi^*\) is the cost of the capital, or the required profitability, the quantity \(\pi^* K(Q)\) is the opportunity cost of capital.\(^{13}\) The pure or economic profit writes as:

---

\(^8\) A very clear synthetic exposé can be found in Tirole 1989, 51-56.

\(^9\) We focus on shareholders and firms: hence, we consider here that the profit comes to remunerate the owners who are the sole last recipients of the residual rights. We let aside the question of other stakeholders not because it is not interesting or important but because the new result concerns shareholders: we follow Descartes’ philosophical method.

\(^10\) Because the output level is the core of the discussion, factors of production are not explicitly considered here: to simplify the exposé, the argument of the maximization program is the output level \(Q\).

\(^11\) See in annex a discussion about the form of the function of capital.

\(^12\) If one wants to consider that the value of capital may change over time, one is able to compute the capital as the average between its value at period’s beginning (i.e., \(ex\ \text{ante}\)) and at the period’s end (i.e., \(ex\ \text{post}\)). Alternately the return of capital may be computed at the beginning of the period and at the end. One may also consider the historic value of the capital. We prefer consider only the value \(ex\ \text{ante}\) or the historic value to avoid complicating the reasoning with financial market considerations.

\(^13\) Following Boulding (1969), calculating the opportunity costs could be difficult. Here, it is the opportunity costs of capital that is in question: indeed, the alternative investment of the funds must be well defined, unique and known, to allow calculating the opportunity cost. Remark that some authors treat the payment of equity as an ordinary cost (Wu 1989, p. 250).
\( V(Q) = \Pi(Q) - \pi^* K(Q) \)

while the profit rate writes as:

\[ \pi(Q) = \frac{\Pi(Q)}{K(Q)} \]

Obviously, if \( K(Q) \) is fixed, \( \max \Pi(Q) \) or \( \max V(Q) \) are equivalent to \( \max \pi(Q) \). However, if an increase of \( \Pi(Q) \) or \( V(Q) \) is obtained by an increase of \( K(Q) \), then both families of programs could diverge. This will be developed now: profit-rate-maximizing firms will be compared to the pure-profit-maximizing firms by computing what the optimal output (and so the optimal capital) which maximizes the pure profit on one hand and the profit rate on the other hand is. To stay inside the limits of a paper, only the two classical polar cases will be examined: monopoly and perfect competition for the single product case.

### 2.1. The optimal outputs

By deriving (1), a pure-profit-maximizing firm reaches is optimum for:

\[ \frac{\Pi'(Q)}{K'(Q)} = \pi^* \]

which is equivalent to \( \Pi'(Q) = \pi^* K'(Q) \iff R'(Q) = C'(Q) + \pi^* K'(Q) \). This expression means that the profit of the marginal unit of output is equal to the cost of the marginal unit of capital. Note that everything is as if the marginal cost \( C'(Q) \) was increased by \( \pi^* K'(Q) \) with an unchanged marginal revenue \( R'(Q) \).

Now, by deriving (2), the condition of optimality for profit-rate maximization is:

\[ \frac{\Pi'(Q)}{K'(Q)} = \pi(Q) \]

which is equivalent to \( \Pi'(Q) = \pi(Q) K'(Q) \iff R'(Q) = C'(Q) + \pi(Q) K'(Q) \). This expression means that the profit of an extra unit of output is equal to the return of an extra unit of capital at the rate \( \pi(Q) \). Everything is as if the marginal cost \( C'(Q) \) was increased by \( \pi(Q) K'(Q) \) with an unchanged marginal revenue \( R'(Q) \).

Both types of companies have a similar equilibrium condition except that \( \pi(Q) \) replaces \( \pi^* \) for the profit-rate-maximizing firm.\(^{14}\) The profit-rate-maximizing firm also produces up to the point where the marginal profit is equal to the average profit multiplied by the relative elasticity of capital; or the profit-rate-maximizing firm produces up to the point where the relative elasticity of the profit is equal to the relative elasticity of the capital:

\[ \Pi'(Q) = e_{k/Q} \Pi(Q) \iff e_{\Pi/Q} = e_{k/Q} \]

in which \( e_{\Pi/Q} \) is the relative elasticity of the profit to the output, \( e_{k/Q} \) is the relative elasticity of the capital to the output and \( \Pi(Q) = \Pi(Q)/Q \) is the profit by unit of output (average profit). By comparison, the pure-profit-maximizing firm produces up to the point where

\[ \Pi'(Q) = e_{k/Q} \pi^* K(Q) \iff e_{\Pi/Q} = e_{k/Q} \frac{\pi^*}{\pi(Q)} \]

---

\(^{14}\) Remark that the condition of entry in the sector for both behaviours, namely, \( V(Q) \geq 0 \) and \( \pi(Q) \geq \pi^* \), are the same, that is, \( \frac{\Pi(Q)}{K(Q)} \geq \pi^* \).
the quantity \( \pi^* K(Q) \) being the cost of capital by unit of output (\( K(Q) = K(Q)/Q \) denoting the capital by unit of output).

### 2.2. Comparison of outputs at optimum

We may compare the optimum of the pure-profit-maximizing firm to those of the profit-rate maximizing firm. Denote \( Q^\Pi \) as the output of a classical pure-profit-maximizing firm at equilibrium (\( Q^\Pi \) is the solution of \( \max_{Q} \Pi(Q) \)) and \( Q^\pi \) as the output for a profit-rate-maximizing firm at equilibrium (\( Q^\pi \) is the solution of \( \max_{Q} \pi(Q) \)).

Theorem 1. Consider the more probable case, \( K'(Q) \geq 0 \) (not less equity to produce more). A profit-rate-maximizing firm has a lower optimal output (and consequently it uses less inputs) than a pure-profit-maximizing firm when the firm is producing, that is, when the profit rate is higher than the cost of capital at the optimum of the profit rate: if \( V(Q^\pi) \geq 0 \iff \pi(Q^\pi) \geq \pi^* \) then \( Q^\pi < Q^\Pi \). If \( V(Q^\pi) < 0 \iff \pi(Q^\pi) < \pi^* \), then \( Q^\pi > Q^\Pi \) but the firm stops producing.

Remark that the case \( \pi(Q^\pi) < \pi^* \) is the less probable than the case \( \pi(Q^\pi) > \pi^* \) because it means that the profit-rate-maximizing firm is less attractive than any alternating investment in the financial market: the firm gets out of the sector. Moreover, one has also \( V(Q^\pi) < 0 \): at the point \( Q^\pi \) the pure-profit-maximizing firm experiences a financial lost and gets out of the sector.

Proof. From (3), the first order condition of optimality to maximize the pure profit \( V \) is: \( \Pi'(Q) - \pi^* K'(Q) = 0 \); consider the curve \( V'(Q) = \Pi'(Q) - \pi^* K'(Q) \). From (4), the first order condition of optimality to maximize the profit rate \( \pi(Q) \) is \( \Pi'(Q) - \pi(Q) K'(Q) = 0 \). Consider the curve \( f_\pi(Q) = \Pi'(Q) - \pi(Q) K'(Q) \). Now let's write \( V'(Q) \) as a function of \( f_\pi(Q) \):

\[
V'(Q) = f_\pi(Q) + [\pi(Q) - \pi^*]K'(Q)
\]

Assume that \( V''(Q) < 0 \) (ordinary second order condition of equilibrium). As \( Q^\Pi \) is the point which maximizes the pure profit, then \( V'(Q^\Pi) = 0 \) and as \( Q^\pi \) is the solution of the maximization of the profit rate, then \( f_\pi(Q^\pi) = 0 \).

1) Suppose that \( K'(Q) \geq 0 \) and \( \pi(Q^\pi) \geq \pi^* \) by assuming \( Q > 0 \). Then from (7) \( V'(Q^\pi) \geq 0 = f_\pi(Q^\pi) \). Near \( Q^\pi \) the curve \( f_\pi(Q) \) is under (or is confused with) the curve \( V'(Q^\pi) \). Thus, \( Q^\Pi \geq Q^\pi \) (and \( p^\Pi \leq p^\pi \) for an ordinary monotonically decreasing demand curve, \( p^\Pi \) and \( p^\pi \) denoting the prices of commodities corresponding to \( Q^\Pi \) and \( Q^\pi \) respectively). To the limit, if \( V'(Q) \) is vertical near \( Q^\Pi \), \( V'(Q) \) will cut the x-axis at \( Q^\pi \) and then \( Q^\Pi = Q^\pi \). See Figure 1.
2) If $K'(Q) \geq 0$ and $\pi(Q^* \geq \pi^*$, the results are reverted and near to $Q^*$, the curve $f_\pi(Q)$ is over curve $V'(Q)$; thus, $Q^\Pi < Q^*$. Remind that in this case, $V(Q^*) < 0$.

Remark that if $K'(Q) \geq 0$ and $\pi(Q^* \geq \pi^*$, and if both $V'(Q)$ and $f_\pi(Q)$ have only one root (“well-behaved” curves), then $V'(Q^\Pi) = 0$ and $f_\pi(Q^\Pi) \leq 0$ at $Q^\Pi$. So from (7), $\pi(Q^\Pi) \geq \pi^* \iff V(Q^\Pi) \geq 0$ must be true if $K'(Q) \geq 0$ (and $\pi(Q^\Pi) < \pi^* \iff V(Q^\Pi) < 0$ if $K'(Q) < 0$): the pure profit of pure-profit-maximizing firms is nonnegative at their optimum, what is a classical result.

Also note that $V'(Q)$ and $f_\pi(Q)$ are not necessarily straight lines (even if $V'(Q)$ is a straight line, $f_\pi(Q)$ is probably not), but if they are, they are not parallel. Their respective slopes would be:

$$V''(Q) = \Pi''(Q) - \pi^* K''(Q)$$

and

$$f'_\pi(Q) = \Pi'(Q) - f_\pi(Q) \frac{K''(Q)}{K(Q)} - \pi(Q) K''(Q).$$

Hence at $Q^*$, $f_\pi(Q^*) = 0$ holds, so $f'_\pi(Q^*) = \Pi'(Q^*) - \pi(Q^*) K''(Q^*)$; if $f'_\pi(Q^*) < V''(Q^*)$,

with $f'_\pi(Q^*) > V''(Q^*)$, $f_\pi(Q^*)$ is necessarily more vertical than $V'(Q^*)$ at the point $Q^*$.

The difference in the outputs can be computed in a particular case by assuming that $V'(Q^*)$ is locally a straight line as in Figure 1. So

$$\frac{V'(Q^*)}{Q^\Pi - Q^*} = \tan \alpha,$$

where $\tan \alpha$ is the absolute value of the slope of $V'(Q)$ at $Q^*$, that is, $\tan \alpha = V''(Q^*)$; hence $Q^\Pi - Q^* = \frac{V'(Q^*)}{V''(Q^*)}$.  

Figure 1. Comparison of optima when $K''(Q^*) > 0$ and $\pi(Q^*) > \pi^*$.
2.3. Applications

In \( \Pi(Q) = R(Q) - C(Q) \), \( R(Q) = p(Q)Q \) is firm’s income, \( p(Q) \) is the inverse function of demand: as the demand is elastic, what is the mathematical general case, the firm is formally a monopoly but the perfectly competitive firm can be derived by considering an inelastic demand. \( C(Q) \) is the cost of production: this one is classically found by minimizing the costs of production for all target output \( Q \) under a technological constraint. It is not necessary to specify immediately what the exact form of the demand is (a decreasing function \( p(Q) \) for monopoly or a constant \( p = \bar{p} \) for perfect competition): the results are valid for monopoly as well for prefect competition.

2.3.1. Monopoly

The above derivations are applicable without changing anything for monopoly but as the demand is a decreasing function, output prices evolve in a reverse way: they are higher and the monopoly power is stronger: if \( \pi(Q^*) > \pi^* \) then \( p^* > p^\Pi \); if \( \pi(Q^*) < \pi^* \), then \( p^* > p^\Pi \).

A particular case of profit-rate-maximizing monopoly is interesting for its simplicity: the fixed coefficient of capital, that is, \( K(Q) = kQ \), with \( k > 0 \) which implies that \( e_{k/Q} = 1 \). From (4) the profit-rate-maximizing monopoly, with a fixed coefficient of capital, produces up to the point where the marginal profit is equal to the average profit (or to the point where the average profit is at a maximum), \( \Pi'(Q) = \pi(Q)k \Leftrightarrow \Pi'(Q) = \Pi(Q) \), that is, \( e_{k/Q} = 1 \); note that everything is as if the marginal cost \( C'(Q) \) was increased by \( \pi(Q)k \) with an unchanged marginal revenue \( R'(Q) \). So, with a fixed coefficient of capital \( k \), a profit-rate-maximizing monopoly produces and invests so that the profit contributed by an extra unit becomes equal to the average profit yielded by other units. By comparison, from (6), the pure-profit-maximizing company with a fixed coefficient of capital produces up to the point where the marginal profit is equal to the cost of capital by unit of output: \( \Pi'(Q) = \pi^*k \) or the company produces up to \( e_{k/Q} = \pi^*/\pi(Q) \). Instead of comparing to \( \pi^*k \) the profit brought by the last unit of output, the \( n \)th, the profit-rate firm compares the profit brought by the last unit of output to the average profit brought by preceding units, \( n-1, n-2, \ldots \), etc. See Figure 2.

We may also consider the Lerner conditions. One finds for the pure-profit maximizing monopoly:

\[
\frac{p(Q) - C'(Q)}{p(Q)} = -\frac{1}{e_{Q/p}} + \pi^* \kappa(Q) e_{k/Q}
\]

where \( \kappa(Q) = \frac{K(Q)}{R(Q)} \) is the capitalistic intensity. The mark-up \( \frac{p(Q) - C'(Q)}{p(Q)} \) is positive as \(-\frac{1}{e_{Q/p}} \) and \( \pi^* \kappa(Q) e_{k/Q} \) are positive (\( K(Q) \) and \( K'(Q) \) are assumed to be nonnegative, hence \( \kappa(Q) \) and \( e_{k/Q} \) are nonnegative): there is monopoly power. And for the profit-rate-maximizing monopoly:

\[
\frac{p(Q) - C'(Q)}{p(Q)} = -\frac{1}{e_{Q/p}} + \pi(Q) \kappa(Q) e_{k/Q}
\]
There is also monopoly power if $\pi(Q)$ is nonnegative. Now, we may compare the Lerner conditions (8) and (9). If $K'(Q) > 0$ then $e_{K/Q} > 0$ and if $K(Q) > 0$ then $\kappa(Q) > 0$. As soon as $\pi(Q) \geq \pi^*$, the mark-up of the profit-rate-maximizing monopoly is higher than the mark-up of the pure-profit-maximizing monopoly; hence the first one has a larger monopoly power with $Q^\pi < Q^\Pi$. If $\pi(Q) < \pi^*$ the mark-up are sorted in reversed order.

![Figure 2. Monopoly with fixed coefficient of capital; the vertical gray lines indicate the mean profits: it is maximum for $Q^\pi$; the hatched rectangles are the pure profits.](image)

### 2.3.2. Perfect competition

We consider now the particular case of perfect competition as an application of the above Theorem 1.

**Corollary 1.** The perfectly competitive firms are smaller when they maximize the profit rate. Consequently, as one may think that they are more numerous: the competition is stronger.

**Proof.** In perfect competition, one has $p = \bar{p}$, $\bar{p}$ being the market price. The optimum of a profit-rate maximizing firm is given by

\[(10) \quad \bar{p}^\pi = C'(Q) + \pi(Q) K'(Q)\]

while for a pure-profit-maximizing firm it is given by $\bar{p}^\Pi = C'(Q) + \pi^* K'(Q)$. If $\pi(Q^\pi) \geq \pi^*$ at optimum, it comes $Q^\pi \leq Q^\Pi$.

In the long-term equilibrium, the pure profit will classically tend to zero, that is, $V(Q) \to 0 \iff \Pi(Q) \to \pi^* K(Q) \iff \pi(Q) \to \pi^*$, what implies also that the profit rate tends to
the required profitability:\footnote{Katzner (2006, p. 553) retrieves this result. It may be noticed that Katzner (2006, pp. 551-557) studies the case of profit-rate maximizing firm but only in perfect competition.} both types of companies have the same long-term equilibrium. Remark that, as firms have a lower output, one can suspect that they are more numerous in the market, so competition could be higher.

Corollary 2. In perfect competition, it is sufficient to have a fixed coefficient of capital (or, more generally, $e_{k/q} = 1$), to do that price plays no role in the equilibrium of the profit-rate-maximizing firm under perfect competition. So, the company produces up to the point where the average cost $\overline{C}(Q)$ is minimum and price is no more a signal: the output does not vary if the price changes and firm’s supply curve is vertical, even we are in perfect competition: inter-firms coordination fails.\footnote{Some cases of coordination failures have been studied in the literature (for example: Heller 1986; Bagwell and Ramey 1994), but they are not comparable.}

Proof. From (10), 
\[ p^* = C'(Q) + \left[ \frac{\pi^* - \overline{C}(Q)}{K'(Q)} \right] Q \] 
where $\overline{C}(Q) = \frac{C(Q)}{Q}$. If $K(Q) = k Q$, then $e_{k/q} = 1$; hence, $C'(Q) = \overline{C}(Q)$: the mean cost is minimum. Notice that the result is $\overline{p} = C'(Q) + \pi^* k$ for the pure-profit-maximizing firm.

2.4. The case of the multiproduct firm

For a multiproduct pure-profit-maximizing monopoly, the pure profit writes as:

\[ V(p, q) = R(p, q) - C(q) - \pi^* K(q) = \sum_{i=1}^{n} p_i(q_i) q_i - C(q) - \pi^* K(q) \]

where $p$ is the output price vector and $q$ is the output vector. I discuss the most probable case: independent demands, dependant costs. \textit{A priori}, there are no reasons to assume that the capital function to be more separable than costs. It comes

\[ \frac{\partial V(p, q)}{\partial q_i} = 0 \iff q_i \left( p_i(q_i) - C'_i(q_i) + p_i(q_i) - C_i(q) - \pi^* K_i(q) \right) = 0 \]

where $C'_i(q)$ and $K'_i(q)$ are the partial derivatives of $C(q)$ and $K(q)$ with respect to $q_i$.

\[ \frac{p_i(q_i) - C'_i(q_i)}{p_i(q_i)} = -\frac{1}{e_{q_i/p_i}} + \pi^* \frac{1}{r_i} \kappa(q) e_{k/q_i} \]

where $e_{q_i/p_i} = \frac{p_i(q_i)}{p_i(q_i) q_i}$ is the elasticity of demand of commodity $i$ by respect to its price, $e_{k/q_i} = \frac{K'_i(q) q_i}{K(q)}$ is the elasticity of total capital by respect to the output of commodity $i$, $\kappa(q) = \frac{K(q)}{R(q)}$ is the intensity of capital and $r_i = \frac{R_i(q_i)}{R(q)}$ (with $R_i(q_i) = p_i(q_i) q_i$) is the share of product $i$ in the total revenue. Obviously, if costs and capital are separable (it is possible to write $C_i(q_i)$ and $K_i(q_i)$ can be identified: $C(q) = \sum_{i=1}^{n} C_i(q_i)$ and $K(q) = \sum_{i=1}^{n} K_i(q_i)$, one
retrieves \( n \) independent monoproduction pure-profit-maximizing monopolies:

\[
\frac{p_i(q_i) - C'_i(q_i)}{p_i(q_i)} = -\frac{1}{e_{q_i/p_i}} + \pi^* \kappa_i(q_i) e_{K_i/q_i},
\]

where \( e_{K_i/q_i} = \frac{K_i'(q_i)}{K(q_i)} \) and \( \kappa_i(q_i) = \frac{K_i(q_i)}{K(q_i)} \).

In case of coefficient of capital, the capital function becomes separable (even if the costs remain dependant): \( K(q) = \sum_{i=1}^{n} k_i q_i \) and \( e_{K_i/q_i} = 1 \); then

\[
\frac{p_i(q_i) - C'_i(q_i)}{p_i(q_i)} = -\frac{1}{e_{q_i/p_i}} + \pi^* \kappa_i(q_i). \quad \text{A special case is those where the required rate of profitability changes with the product:}
\]

\[
V(p, q) = \sum_{i=1}^{n} p_i(q_i) q_i - C(q) - \sum_{i=1}^{n} \pi^*_i K_i(q_i); \quad \text{then}
\]

\[
\frac{p_i(q_i) - C'_i(q_i)}{p_i(q_i)} = -\frac{1}{e_{q_i/p_i}} + \pi^*_i \kappa_i(q_i) e_{K_i/q_i}; \quad \text{but this requires the function of capital to be separable.}
\]

For a multiproduct profit-rate-maximizing monopoly, the profit-rate writes as:

\[
\pi(p, q) = \frac{R(p, q) - C(q)}{K(q)} = \frac{\sum_{i=1}^{n} p_i(q_i) q_i - C(q)}{K(q)}
\]

under the same hypotheses about demands, costs and capital. It comes:

\[
\frac{\partial \pi(p, q)}{\partial q_i} = 0 \iff \left[ q_i \left( p_i'(q_i) + p_i(q_i) - C'_i(q_i) \right) K(q) - \left[ \sum_{i=1}^{n} p_i(q_i) q_i - C(q) \right] K_i(q) \right] K_i'(q) = 0
\]

(12) \( \iff \frac{p_i(q_i) - C'_i(q_i)}{p_i(q_i)} = -\frac{1}{e_{q_i/p_i}} + \pi(q) \frac{1}{r_i} \kappa(q) e_{K_i/q_i}, \)

Again if costs and capital are separable, one retrieve \( n \) independent monoproduction profit-rate-maximizing monopolies:

\[
\frac{p_i(q_i) - C'_i(q_i)}{p_i(q_i)} = -\frac{1}{e_{q_i/p_i}} + \pi_i(q_i) \kappa_i(q_i) e_{K_i/q_i}, \quad \text{where} \quad \pi_i(q_i) = \frac{\Pi_i(q_i)}{K_i(q_i)}.
\]

And in case of coefficient of capital with dependant costs,

\[
\frac{p_i(q_i) - C'_i(q_i)}{p_i(q_i)} = -\frac{1}{e_{q_i/p_i}} + \pi_i(q_i) \frac{1}{r_i} \kappa_i(q_i).
\]

The comparison of optimal outputs is similar than for the monoproduction case: as equations (11) and (12) are identical except that \( \pi(q) \) replaces \( \pi^* \) in (12) the mark-up is higher for the profit-rate monopoly than for the pure-profit monopoly because \( \pi(q) \geq \pi^* \) (if not, the firm stops producing).\(^{17}\)

### 2.5. Specifying what the capital is

We can examine the cases where the capital \( K(Q) \) is the capital engaged (equity plus debts) and where it is the equity only. Denote \( E(Q) \) the equity, \( D(Q) \) the debts, \( K^T(Q) \) the capital engaged, with \( K^T(Q) = E(Q) + D(Q) \). \( C^p(Q) \) the cost of production, \( \Pi^p(Q) \) the

\(^{17}\) Unless the capital function is separable, the firm does not know what product to stop.
production (or operating) profit,\textsuperscript{18} $\Omega(Q) = \Pi^p(Q) - d \cdot D(Q) = p \cdot Q - [C^p(Q) + d \cdot D(Q)]$ the net profit, $e$ the fixed cost of equity (COE),\textsuperscript{19} $d$ the fixed cost of debts, $\delta = D(Q)/K(Q)$ the debt to capital ratio: we assume that $\delta$ is fixed, in order to avoid dealing with the leverage effect;\textsuperscript{20} hence, $w = e \cdot (1 - \delta) + d \cdot \delta$ is the WACC (Weighted Average Cost Of Capital), fixed. We will see that the pure profit (EVA) does not depend on what the capital is.

### 2.5.1. $K(Q)$ as capital engaged

If $K(Q)$ is the capital engaged, $K(Q) = K^T(Q)$; then $\Pi(Q) = \Pi^p(Q)$, $C(Q) = C^p(Q)$, $\pi^* = w$. The pure or economic profit becomes:\textsuperscript{21}

\begin{equation}
V(Q) = \Pi^p(Q) - w \cdot K^T(Q)
\end{equation}

The corresponding profit rate is the ROCE (Return on Capital Employed, as termed by corporate finance) $\eta$.\textsuperscript{22} $\pi(Q) = \eta(Q)$ with

\begin{equation}
\eta(Q) = \frac{\Pi^p(Q)}{K^T(Q)}
\end{equation}

Now, Theorem 1 can be directly applied in a corollary. Denote $Q^\Pi$ as the output of a classical pure-profit-maximizing firm at equilibrium ($Q^\Pi$ is the solution of $\max_V V(Q)$) and $Q^\pi$ as the output for a profit-rate-maximizing firm at equilibrium ($Q^\pi$ is the solution of $\max_Q \eta(Q)$).

Corollary 3. Consider the more probable case, $K^{\pi}(Q) \geq 0$ (to produce more, the firm cannot use less capital engaged). A profit-rate-maximizing firm has a lower optimal output (and consequently it uses less inputs) than a pure-profit-maximizing firm when the firm is producing, that is, when the profit rate is higher than the WACC at the optimum of the profit rate: if $V(Q^\pi) \geq 0 \iff \eta(Q^\pi) \geq w$ then $Q^\pi < Q^\Pi$. If $V(Q^\pi) < 0 \iff \eta(Q^\pi) < w$, then $Q^\pi > Q^\Pi$ but the firm stops producing.

\textsuperscript{18} The idea of operating profit is close to the EBIT (Earnings Before Interest and Taxes, as termed in corporate finance theory) or the NOPAT (Net Operating Profit After Taxes) according to whether taxes are included or not. We consider also that the operating profit is net exceptional charges, etc.

\textsuperscript{19} The cost of equity includes the risk premium by the classical formula of the CAPM model (valid for a diversified portfolio): $r^* = r^{RF} + \beta (r^M - r^{RF})$, where $r^{RF}$ is the risk-free rate to pay the equity, $r^M$ is the market rate, $r^M - r^*$ is the market-risk premium and $\beta$ is the sensibility coefficient measuring the volatility of firm's securities. Other more complicated models, as APT, are also suitable.

\textsuperscript{20} We assume also that Fisher’s separation theorem holds: the financing decisions are independent to consumption decisions if the financial markets are perfect. Hart (1979) have examined incomplete markets.

\textsuperscript{21} It is the EVA (the Economic value Added, following the terminology of Stern Stewart & Co). The economic profit is sometimes criticized (Aglietta and Reberioux 2005), mainly because it uses $w$, or $e$ and $d$ if one prefers, a ratio that includes expectations.

\textsuperscript{22} The ROCE must not be confused with the return on assets (ROA), which has the same numerator but a different denominator, the total assets, while the total net assets (= total assets minus total liabilities) are in the denominator of ROCE.
2.5.2. \( K(Q) \) as equity

For the next section, that deals with shareholders’ behaviour, it is useful to transpose them, above results, *mutatis mutandis*, to the case where the equity is considered instead of the capital engaged, even if a function of equity capital may seem more discusssable than a function of capital engaged, because of the leverage effect that encourage to substitute the debts to the equity.\(^{23} \) We will see now that the results are largely unchanged.

If \( K(Q) \) is the equity, \( K(Q) = E(Q) \), \( \Pi(Q) = \Omega(Q) \), \( C(Q) = C^p(Q) + d \ D(Q) \), \( \pi^* = e \).

The pure profit writes as:

\[
V(Q) = \Omega(Q) - e \ E(Q)
\]

It is elementary to prove that \( \Omega(Q) - e \ E(Q) = \Pi^p(Q) - \pi^* \ K^T(Q) \). The corresponding profit rate is the ROE (Return On Equity): \( \pi(Q) = \frac{\Omega(Q)}{E(Q)} \) with

\[
r(Q) = \frac{\Omega(Q)}{E(Q)}
\]

Now, the following corollary ensues directly from Theorem 1; it will be useful in the next section. Denote \( Q^{\Pi} \) the solution of \( \max_{\hat{Q}} V(Q) \) and \( Q^* \) the solution of \( \max_{\hat{Q}} r(Q) \).

**Corollary 4.** Consider the more probable case, \( E'(Q) \geq 0 \) (not less equity to produce more). A profit-rate-maximizing firm has a lower optimal output (and consequently it uses less inputs) than a pure-profit-maximizing firm when the firm is producing, that is, when the profit rate is higher than the cost of equity at the optimum of the profit rate: if

\[
V(Q^*) > 0 \Leftrightarrow r(Q^*) > e \quad \text{then} \quad Q^* < Q^{\Pi}.
\]

If \( V(Q^*) < 0 \Leftrightarrow r(Q^*) < e \), then \( Q^* > Q^{\Pi} \) but the firm ends.

2.5.1. Equivalence between both objective

Maximizing the ROE \( \eta(Q) = \frac{\Pi^p(Q)}{K^T(Q)} = \frac{\Pi^p(Q)}{E(Q) + D(Q)} \) is the same thing than maximizing the ROE \( r(Q) = \frac{\Pi^p - d(Q)D(Q)}{E(Q)} \) as soon as \( \delta \) is fixed: \( r(Q) = \frac{\eta(Q)}{1-\delta} - d \ \frac{\delta}{1-\delta} \); then \( \eta(Q) = (1-\delta) r(Q) + \delta d(Q) \).

\(^{23} \text{It is very known that, in first approximation, one is able to increase the return on equity (ROE) by appealing to debts, because of the financial leverage effect taking place between ROE and the return on capital employed (ROCE). Fortunately, modern finance theory admits that if a firm is deeply indebted, the risk increases, the minimal required profitability of debts grows: this will compensate the benefit of a call to debts. The Modigliani-Miller's theorem teaches us that firm's value does not depend on the financial structure when the financial markets are perfect: the leverage effect is an illusion and plays no more than a temporary role, that is, only when the firm wanders from equilibrium and before a further return to equilibrium.} \)
3. The switching between companies’ microeconomic behaviours

All the above results are not a class-room exercise! It remains to determine under what conditions a profit-rate objective is chosen by the firm. The question is: why switching from one behaviour –pure profit maximization– to the other –profit rate maximization–?

Voluntarily remaining inside the neoclassical corpus, we will focus on one particular reason (we do not pretend that all reasons are captured in this paper) that can make the firm to switch from one behaviour to the other: its ownership. Working on the ownership is very classical but we will see that it could still serve to produce an interesting explanation. Generally, scholars consider that the main argument really discriminating between profit maximization and other behaviours is the owner's behaviour argument. Firm’s behaviour depends on shareholders-owners’ objective in the following way, leaving aside any questions like risk aversion, agency theory, corporate governance, residual rights of stakeholders other than shareholders, etc. When he owns the firm, the shareholder wants to maximize its utility, which depends of its income. As the profit is included in owner's income, when maximizing its utility, the owner wishes the company to maximize the profit and, as owner's behaviour is supposed to influence firm's behaviour, the company does maximize the profit.

However, scholars focus the analysis on the bipolar relation between the owner and the firm, what can be called a micro-relation: it is the case of agency theory. Here we will analyze the relation between all owners and all firms, what can be called a macro-relation. Usually there is also an implicit hypothesis: each shareholder owns a fixed proportion of each firm's equity, what implies that owners always tend to maintain their control ratio over the company so they adopt what I call a strategic behaviour. However, a dual behaviour is also possible: the owners may maintain fixed the proportion of each firm in their portfolio what mechanically leads to what I call a financial behaviour.

In the following, the index $i$ refers to the shareholders and the index $j$ to the firms. $E_j$ denotes the value of equity capital of company $j$; $D_j$ denotes its debts. $\Pi_j^o$ stands for the operating (or production) profit; it is net of taxes, exceptional charges, etc. The net profit of company $j$ is denoted $\Omega_j$: $\Omega_j = \Pi_j^o - d_j D_j$ where $d_j$ denotes the cost of debts of firm $j$, assumed fixed; $e_j$ denotes the cost of equity (COE) required or expected by shareholders for firm $j$’s shares, assumed fixed.

Theorem 2. If the macro-relation between all shareholders and all firms is considered, apart from agency problems,

- If shareholders have a strategic behaviour, the firms maximize their pure profit (i.e., their EVA)
- If shareholders have a financial behaviour, the firms maximize their profit rate, the ROE (or the ROCE if the cost of debts and the debt to capital ratio are fixed).
- If shareholders have a sleeping-partner behaviour, the firms to maximize their profit rate, the ROE (or the ROCE if the cost of debts and the debt to capital ratio are fixed) but only ex ante.

---

24 Managers cannot do everything as if shareholders do not exist: even when the equity represents a low part of total financing by respect to loan and self-financing, if the price of firm’s share falls, managers do their best to make the price going up. However, it is not true that all shareholders have an influence over the firm: individual shareholders have a low influence, even if they are employed by the firm.
The demonstration of this theorem is given in what follows.

### 3.1. The traditional economic-profit maximization: the strategic behaviour

Denote $\Omega^i_j$ the net profit distributed by company $j$ to shareholder $i$ and $\Omega'$ the total net profits received by the shareholder $i$ (or stock-exchange income):

\[
\Omega' = \sum_j \Omega^i_j
\]

Denote by $\theta^i_j = E^i_j / E_j$ the control ratio of $i$ over $j$, that is, proportion of company $j$'s equity owned by any shareholder $i$, with $\sum \theta^i_j = 1$, where $E_j$ denotes the value of the total equity capital of firm $j$, and $E^i_j$ is the value of the equity capital invested by owner $i$ in company $j$.

If the control ratios $\theta^i_j$ are fixed, that is, if $i$ owns a fixed proportion of $j$'s equity, the profit is distributed proportionally to the equity capital: $\Omega^i_j = \theta^i_j \Omega_j$.\(^{25}\) From (17), the stock-exchange income of shareholder $i$ writes as:

\[
\Omega' = \sum_j \theta^i_j \Omega_j
\]

Any shareholder $i$ tends to prefer the largest stock-exchange income as possible, that is:

\[
\max_{\Omega_j} \left( \Omega_1, \ldots, \Omega_i, \ldots, \Omega_n \right) \Omega' = \sum_j \theta^i_j \Omega_j
\]

of which solution is found when each $\Omega_j$ is maximum; this implies that firms maximize their net profit $\Omega_j$ (leaving aside the questions of corporate governance, agency problems, etc.).

Equivalently, by considering that the price $p_j$ of $j$'s share is equal to the value of $j$'s equity, $E_j$, divided by the number of $j$'s shares, $N_j$ (that is, $p_j = E_j / N_j$), what implies that the number of shares $j$ owned by $i$, $N^i_j$, is equal to $E^i_j / p_j$, it would have been equivalent to consider that any shareholder $i$ maintains fixed the proportion of shares of company $j$ that $i$ earns because (18) is equivalent to

\[
E' = \sum_j N^i_j / N_j \Omega_j
\]

with $\theta^i_j = N^i_j / N_j$: $\theta^i_j$ is not affected by the price variation of firm $j$'s share.

As the parameter $\theta^i_j$ is the control ratio of the shareholder $i$ over the company $j$, assuming $\theta^i_j$ to be fixed means that the shareholder $i$ maintains its control ratio at a fixed level: its behaviour is strategic. However, the shareholders have a strategic behaviour also because they tend to answer positively to all new issues of shares. The fixity of $\theta^i_j$ implies that a strategic shareholder $i$ will subscribe to all future issues of new shares by each company $j$ (as soon as $\theta^i_j > 0$) to maintain stable its $\theta^i_j$. For example, if shareholder $i$ owns 15% of firm $j$ (for example $30M$ on a total of $200M$) and if company $j$ issues $10M$ of new equity capital, shareholder $i$ will have to buy $1.5M$ of these new shares to keep its percentage to 15%.

\(^{25}\) The funds kept in reserve by the firm are neglected.
The category of strategic shareholders is very large: they can be individual capitalists, families of heirs acting as a closed club, or large companies which want to maintain their control, even partial, on a firm: even state owned companies can have this behaviour in countries where a large part of industry remains in public hands. But they have always a strategic reasoning (having the majority of control, or the blocking minority, or having a right of inspection, or being a partner in a joint-venture, etc.). For example, a majority owner, at 51%, will obviously try to maintain this ratio but it is the same for a shareholder with the blocking minority. Other smaller shareholder could or could not maintain its ratio: it depends on what its strategic behaviour is. Sometimes, an owner with 5% will think that it is important to maintain his sharing because he can have an administrator, be well informed on the firm or even be the leader of the Board even with this low percentage. Remark that, by respect to the portfolio theory, shareholder $i$'s portfolio may be under optimal when $i$ has a strategic behaviour: if for at least one company $j$, a shareholder $i$ has a strategic behaviour, the corresponding control ratio $\theta_j^i$ could be higher (the most probable case) or lower than what is recommended by the portfolio theory.

Now we have to consider the opportunity cost of equity. In formula (18), company $j$ distributes its net profit $\Omega_j$, what leads owners to choose as behaviour the maximization of the net profit. We remark that the economic profit is not yet the objective of shareholders or companies but the objective of maximal net profit cannot be exactly those of firms because one must take into account the opportunity cost of company $j$'s capital, $e_j E_j$, leading to the maximization of the economic profit. The quantity

$$V_j' = \Omega_j - e_j E_j$$

is firm $j$'s economic profit equal to the net profit minus the opportunity cost of equity or EVA. As the owner endures an opportunity cost $e_j \theta_j^i E_j$ on its capital as soon as any alternative investment on the financial market is possible, he should not wish company $j$ to maximize directly the net profit, $\Omega_j$. He must subtract this opportunity cost from its income by considering the “pure” dividend, that is:

$$V_j' = \Omega_j - e_j \theta_j^i E_j = \theta_j^i V_j$$

Finally, as the economic profit replaces the net profit in (18) when the opportunity cost of equity is taken into account, any shareholder $i$ maximizes its pure stock-exchange income

$$V_i' = \sum_j V_j'$$

$$\max V_i'(V_1,...,V_j,...,V_m) = \sum_j \theta_j^i V_j$$

and the firms are conducted to maximize also their pure profit $V_j$. However, it is the net profit that is distributed to owners, not the economic profit, because the opportunity cost of equity capital is not an explicit cost and the corresponding money must return to owners: this money is actually distributed and not retained by the company. Otherwise, owners would support a true cost on their capital because they would never be fully paid: the equity capital $e_j E_j$ remains effectively paid to owners by firm $j$ because the set of owners have endured an opportunity cost on it. These results are classical in the context of the microeconomic theory.
3.2. Return-on-capital employed as alternative: the financial behaviour

Fixity of $\theta_i$ is not a general rule because many owners could not want to exert a strategic control on the firm. This situation will be studied in what follows. Shareholders are now assumed to be concerned only with the composition of their own portfolio, taking the percentage of their own equity invested in each company as the command variable, instead of keeping stable the proportion of firm’s equity owned by each shareholder as for strategic shareholders. In other words, shareholders now think in terms of optimal portfolio — probably following the teachings of portfolio theory — rather than in terms of control of firms: their behaviour is financial rather than strategic. They can be professional investors, pension funds, companies, open-ended investment trust, etc.

So, one can introduce the composition of shareholders’ portfolio as the main parameter of the problem by considering $\lambda_j = E_j / E^i$ as the fraction of the equity capital of shareholder $i$ that he has invested in firm $j$ (one has $\sum_j \lambda_j = 1$ for all $i$). The matrix $\Lambda$ of the $\lambda_j$ is considered as a matrix of coefficients, fixed in the short term; now, shareholders’ behaviour is very different from (22). It becomes possible to derive a new expression for the profits paid to any shareholder $i$, resuming from (18):

$$\Omega_i = \sum_j E_j \Omega_j = \sum_j E_j \frac{\Omega_j}{E_j} = \sum_j \lambda_j r_j$$

where $r_j = \Omega_j / E_j$ stands for firm $j$’s ROE and $\lambda_j = E_j / E^i$ for all $i, j$ are assumed fixed. As shareholder $i$ maximizes its net stock-exchange income $\Omega_i$:

$$\max_{r_j} \Omega_i (r_1, ..., r_j, ..., r_m) ; \Omega_i = E^i \sum_j \lambda_j r_j$$

any firm $j$ maximizes its ROE $r_j$. By dollar of invested equity capital, any financial shareholder $i$ maximizes a weighted sum of $r_j$. The quantity $r' = \sum_j \lambda_j r_j$, where $r'$ denotes the ratio $\Omega / E^i$, can be called the weighted mean ROE of shareholder $i$; hence a “financial shareholder” maximizes a weighted mean of ROE: at each instant, by assuming that the $\lambda_j$ are fixed (before deciding to change its portfolio), a shareholder $i$ wants the company $j$ to maximize its ROE $r_j$. Again leaving aside the question of corporate governance, one could assume that this implies the firm to maximize itself the same objective. Remark that it is not necessary to assume that $E^i$ is fixed because this term affects equally all firms $j$ from the point-of-view of any shareholder $i$. Moreover, reasoning here in terms of number of shares is nonsense: the ratio $E^i / E^i$ takes sense only in value and never in terms of number of shares and the ratio $N^i_j / E^i$ is nonsense.

Obviously, when the price $p_j$ of company $j$’s shares changes, mechanically, the invested capital $E^i_j$ varies proportionally in the same direction, so $E^i = \sum_j E^i_j$ varies also and the ratio $\lambda_j$ evolves: it is sensitive to share prices, that is, seemingly unstable. However, any financial shareholder $i$ takes this into account and he adapts its own portfolio following the portfolio theory: he chooses its vector $(\lambda_1, \lambda_2, ..., \lambda_m)$ exogenously following the teachings of portfolio theory. The financial practices always think in terms of diversified
portfolio, following the classical portfolio theory (for example, they could advise clients to buy x% of US shares, y% of European shares, z% of Pacific shares, etc.). Shareholders choose an optimal combination of securities on the efficient market curve which indicates the optimal combination of the expected profitability of the whole portfolio and its risk. On the other hand, if they want to minimize their risk, shareholders must diversify their portfolio up to obtain a miniature representation of the market. Overall, the $\lambda_j$ must be considered as exogenous, that is, fixed up to the moment of choosing a new vector.

Now, if the opportunity cost of equity capital is taken into account, the economic profit replaces the net profit in (23); let’s demonstrate it. Any financial shareholder maximizes $V^i = \sum_j \frac{E^i_j}{E_j} V_j = E^i \sum_j \frac{E^i_j}{E^i} \frac{V_j}{E_j} = E^i \sum_j \lambda_j v_j$, where $v_j = \frac{V_j}{E_j}$ is the “pure-profit rate”. The shareholder $i$ is conducted to solve:

$$\max_{v_j} V^i(v_1, \ldots, v_j, \ldots, v_m) V^i = E^i \sum_j \lambda_j v_j$$

hence, any shareholder $i$ now wants the firm to maximize the “pure-profit rate” $v_j$. Fortunately, both maximization programs obtained from both types of ROE, without and with opportunity costs — $r_j$ on one hand and $v_j$ on the other hand— give the same solution (the variable being for example the output $Q$) because $v_j$ is only a translation of $r_j$:

$$v_j = \frac{\Omega_j - e_j}{E_j} = r_j - e_j$$

hence, $\max v_j \Leftrightarrow \max r_j$. Finally, remind that maximizing the ROCE is the same thing than maximizing the ROE as soon as $\delta_j$ is fixed: $\eta_j = r_j (1 - \delta_j) + d_j \delta_j$ and $\max \eta_j \Leftrightarrow \max r_j$.

The prototype of the firm having such a financial behaviour is the conglomerate, also called financial group. It often uses the ROE, or the return-on-invested-capital, as indicator to decide which company is destined to stay inside the conglomerate and which is to get out or get in: if they are under a desired rate, subsidiaries are sold out, unless it is possible to put them above the desired rate in a few months by some radical measures as massive redundancy. Historically, ITT under the management of Harold Geenen was the best example of this behaviour. General Electric, the largest conglomerate (and the largest company) in the world, has never been managed like that in the past; however, it has announced that it will sold the household equipment division to increase its ROCE.

It must be noticed is that, in business practice, ROE or ROCE are used to evaluate the performance of the firm but are never considered as an objective to be maximized. The ratios of profitability are taken as a minimum which must be reached by firms but not that must be maximized. However, unlike what is often asserted (see for example Plummer, Sheppard and Haining, 1998), maximizing the profit rate is not only a Marxian objective: the archetype of capitalist behaviour leads to the same goal!

### 3.3. The sleeping-partner behaviour

Shareholders could think that it is not so important to maintain their sharing fixed when this sharing, very low, does not allow any strategic control. As an alternative

---

26 By introducing a non-risky asset, one obtains the capital-market line which indicates the expected profitability of a portfolio combining a non-risky asset and the market (risky) asset. The tangency point between the curve and the line is the market portfolio.
behaviour, shareholders $i$ can also be assumed to own a fixed quantity of firm $j$’s capital: the investment $E_j$ of any shareholder $i$ in any company $j$ is maintained fixed. From (18) it comes naturally, excluding opportunity costs of equity: $\Omega_i = \sum_j E_j \frac{\Omega_j}{E_j}$, that is, any shareholder $i$ solves:

$\text{(27)} \quad \max_{r_j} \Omega_i (r_1, \ldots, r_j, \ldots, r_m), \quad \Omega_i = \sum_j E_j r_j$

what implies that any firm $j$ maximizes its ROE $r_j$.

With opportunity costs of equity, any shareholder $i$ has to maximize $V_i = \sum_j E_j \frac{V_j}{E_j}$, that is,

$\text{(28)} \quad \max_{V_j} \left\{V_i, \ldots, V_j, \ldots, v_m\right\}V_i = \sum_j E_j v_j$

where $v_j = V_j / E_j$ and any firm $j$ maximizes its “pure-profit rate” $v_j$. Both expressions (27) and (28) lead to the same maximization—as for the financial shareholders—of the ROE $r_j$ by each company $j$, what is equivalent to ROCE maximization if $d_j$ and $\delta_j$ are fixed.

Here, it is equivalent to consider that any shareholder $i$ maintains fixed the number of $j$’s shares that he earns because (27) is equivalent to $E_i = \sum_j N_j E_j / N_j$ where $E_j / N_j$ is the dividend by share: $i$ will not subscribe to new issues of shares of $j$. This is the behaviour of the family man or of a sleeping partner: an owner $i$ buys directly a number $N_j$ of shares of a company $j$ and he keeps these shares indefinitely, perhaps forgetting them. This is also the behaviour of the employee-owner, but, as underlined by Milgrom and Roberts (1992, p. 40), an employee-owner will also demand better salaries what is contradictory with both pure-profit and profit-rate maximization. Ironically, the sleeping-owner behaviour is similar to those of the financial shareholders, even if the stability of sleeping partnership is very high in the long term. Nevertheless, sleeping-partner shareholders have a small influence on the firm, precisely because they are sleeping partners. One cannot argue, only by looking at equations (27)-(28), that they influence the firm and oblige it to maximize the ROE or ROCE. This is because, even if the firm does not respect their wish, they will not tend to get out; this wish was perhaps rather strong when they have bought the shares but later, two or three years, sometimes much more, they never think to ask for a better profitability. Hence it is only ex ante that the sleeping partner may have an influence.

Moreover, how many family men exist in the business world? Quantitatively, many, but they do not represent a large proportion of companies’ shareholding measured by the ratio $N_j / N_j$: each family man owns a very small number of shares and ordinary people often prefer not to be directly shareholders and consider investing in open-ended investment trust

---

27 Financial analysts consider that it is necessary to keep the direct shares in the portfolio for a long period, at least five years, to avoid losses in capital: this is why the sleeping-partner shareholding is so stable. Remark that Dewatripont and Tirole (1994) have argued that equity-holders are passive in the control of the firm while debt-holders are active; for Berglõf and Thadden (1994) short-term debt-holders are active but long-term debt-holders are passive. For a synthetic discussion on this particular point, see Hart (1995), chap. 5.
managed by professional investors. In the privatization programs conducted by some governments in Europe, Western and Eastern, a small but significant proportion of the shareholding is reserved for the public; generally, this public issue is rapidly exhausted.\footnote{In privatization programs, the shares are sometimes sold at an attractive price (e.g. 10\% under the true value) to be sure that all of them will be rapidly sold out. Hence, there is the possibility of a hit-and-run to catch an immediate gain in capital: this is a speculative behaviour allowed by the government’s gift to the public.}

### 3.4. Discussion

#### 3.4.1. Financial shareholders versus strategic shareholders

What could mean the opposition between financial shareholders and strategic shareholders? Strategic shareholder firms could be large companies simply wanting to maintain the ownership at a given level, 100\%, 51\%, 33\%, etc. Strategic-shareholder firms are not necessarily old companies, out-fashioned, family-owned: they could be large and dynamic groups; simply, the ownership is stable and faithful, what allow the firms to maximize the profit. On the other hand, when a company has a financial ownership in the sense defined in this article, companies are not necessarily more dynamic. Simply, as the investors have chosen a dynamic management of their own portfolio (by maximizing profitability) and as they have these companies on their list of possible investment, these firms cannot be sure of the faithfulness of their shareholders. This obliges these firms to maximize their ROE or ROCE in order to keep their shareholding as stable as possible, to avoid price share going down, leading them to produce a lower output, using fewer inputs and specially labour (even if these companies are not necessarily less extensive in labour).

#### 3.4.2. Simultaneous stability of $\theta$ and $\lambda$

If we denote $E$ the matrix of terms $E^j_i$, with shareholders by rows and firms by columns, defining coefficients $\theta^j_i$ in the in strategic behaviour means that $E$ is read by columns while in the financial behaviour, defining coefficients $\lambda^j_i$ means that matrix $E$ is read by rows. The simultaneous stability of the coefficients $\theta^j_i$ and $\lambda^j_i$ is impossible because of the following property:

**Property 1.** It is impossible to assume that both company's capital structure $\theta^j_i$ and shareholder's portfolio structure $\lambda^j_i$ are fixed at the same time, for all couples $(i, j)$, unless the cross structure of capital $E^j_i / E^i$ to be fixed or the capital varies homothetically in $i$ and $j$. Both conditions are not credible: there is no reason to assume that the equity capital $E^j$ of any firm $j$ varies proportionally to the equity capital $E^i$ invested by any shareholder $i$ or that the capital of company $j$ evolves as those of shareholder $i$.

**Proof.** From their definition, the following relation between $\theta^j_i$ and $\lambda^j_i$ always holds:

$$\lambda^j_i = \theta^j_i \frac{E^j_i}{E^i}.$$  

So, if shareholders' portfolio coefficients $\lambda^j_i$ are assumed fixed, this implies that the firms' capital structure $\theta^j_i$ cannot be fixed and conversely. Consider an *ex ante* situation...
with $E^i$ and $E_j$ and consider an \textit{ex post} situation where the equity capital has varied becoming $\tilde{E}^i$ and $\tilde{E}_j$ (where tilde denotes the new quantity): $\lambda^i_j = \theta^i_j \frac{E^i_j}{E^i}$ and $\lambda^i_j = \tilde{\theta}^i_j \frac{\tilde{E}^i_j}{E^i}$. Assume that $\theta^i_j$ is fixed: $\theta^i_j = \tilde{\theta}^i_j$; so $\lambda^i_j = \theta^i_j \frac{E^i_j}{E^i} = \lambda^i_j$. Conversely, if we assume that $\lambda^i_j$ is fixed, $\lambda^i_j = \tilde{\lambda}^i_j$, so $\tilde{\theta}^i_j = \lambda^i_j \frac{\tilde{E}^i_j}{E^i} \equiv \theta^i_j$. Simultaneous stability would imply that the cross structure of capital $\frac{E^i_j}{E^i}$ to be fixed, that is, $\frac{\tilde{E}^i_j}{E^i} = \frac{E^i_j}{E^i}$ for all $i, j$ or that the capital evolves homothetically in $i$ and $j$, that is, $\frac{\tilde{E}^i_j}{E^i} = \frac{E^i_j}{E^i}$ for all $i, j$.

Property 1 prevents the behaviour to be mixed for a given couple $(i, j)$ but it does not prevent the behaviour to be mixed inside a same company $j$. Technically this means that the matrix $E$ is partitioned in two blocs, after appropriate sorting, that is, $\begin{bmatrix} sE \\ fE \end{bmatrix}$, where $sE$ is the bloc of strategic shareholders that serves to define the $\theta^i_j$ and $fE$ the bloc of financial shareholders that serves to define the $\lambda^i_j$, each $E_j$ being separated into two parts: $E_j = sE_j + fE_j$, where $s$ and $f$ are the indexes for the strategic shareholders and the financial shareholders respectively. Equation (18) becomes $\Omega_i = \sum_{j} \frac{sE^i_j}{E^i} \Omega_j = \sum_j \frac{sE^i_j}{E^i} \Omega_j$ with $\frac{sE^i_j}{E^i} = \gamma_j \sum_{j=1}^{S} \frac{sE^i_j}{E^i}$ where $\sum_{i=1}^{S} \frac{sE^i_j}{E^i} = \gamma_j$ (and $S$ is the number of strategic shareholders) with $\gamma_j \in [0,1]$; equation (23) turns out into $\Omega_i = \sum_{j} \frac{fE^i_j}{E^i} \Omega_j = \sum_j \frac{fE^i_j}{E^i} \Omega_j = \frac{fE^i_j}{E^i} \sum_j \frac{fE^i_j}{E^i} \Omega_j = \frac{fE^i_j}{E^i} \sum_j \frac{fE^i_j}{E^i} \lambda_j r_j$ with $\frac{fE^i_j}{E^i} = \sum_{j} \frac{fE^i_j}{E^i} = 1$. The mixed behaviour is examined now.

### 3.4.3. Firms’ mixed behaviour

A company $j$ which has a capital owned by a closed group of strategic shareholders has no incentive to maximize its ROE because the coefficients $\theta^i_j$ are fixed: this firm must maximize classically its EVA. So, what happens if a company, which has a closed ownership, opens it to any other type of shareholders? This firm could have to switch its behaviour from EVA to ROE, at least regarding these new shareholders. If the shareholding switches from a strategic behaviour to a financial behaviour, firm’s behaviour will switch from economic-profit maximization (EVA) to profit-rate maximization (ROE or ROCE). Hence, the ownership may be also mixed. Sometimes, some companies have a core of strategic shareholders, and outside it, a set of shareholders who have a financial behaviour or a sleeping-partner behaviour. In this case, the firm receives an incentive from strategic shareholders to maximize EVA and at the same time an incentive to maximize ROE-ROCE.
from financial and sleeping-partner shareholders: if $\gamma_j$ is the proportion of company's capital owned by strategic shareholders, for a firm $j$, the behaviour simply becomes a linear combination of both objectives:

$$\max \left[ \gamma_j, V_j + (1 - \gamma_j)\pi_j \right]$$

Program (29) holds if both groups of owners are assumed to have an influence over the firm in proportion to $j$'s capital that they owned; if it is not the case, the parameter $\gamma$ must reflect this influence and not the true proportion. Theorem 3. An economic-profit-maximizing company which accepts to give stock-options or other forms of profit-sharing to its high management could partially abandon EVA-maximization for ROE-maximization.

Proof. When a manager $i$ receives stock-options of a company $j$, he becomes an owner of the firm but with a sleeping-partner behaviour because the amount $E^0_j$ or $N^0_j$ is fixed for years and not the proportions $\theta_j$ or $\lambda_j$. When the option is released, the shareholder adopts a financial behaviour because its control rate remains generally weak. So, in economic-profit-maximizing firms, the manager has an incentive to maximize ROE instead of the economic profit as soon as he receives and buys stock-options.

3.4.4. Natural selection argument

Natural selection became very popular after Nelson and Winter's book (1982); nevertheless, it was previously developed. For Alchian, Darwinian selection allows some companies, coming closer to a great profit, to survive better than the others, going bankrupt (Alchian 1950); this argument was also developed by Friedman (1953). The existence of such a mechanism is classically assumed to be sure that profit maximization is the function actually chosen by the firm (Machlup 1946). Koopmans (1957) has criticized this approach by arguing that if natural selection explains profit maximization then the natural selection must be postulated, not profit maximization; Schaffer (1989) or Beker (2004) have also exposed a point of view partially contradictory to natural selection.

What natural selection has to see with the pure profit and profit-rate-maximization? In the normal case where $K_j'(Q^*_j) > 0$ and $\pi_j(Q^*_j) > \pi_j$, for a firm $j$, denote by $\pi_j^*$ the value of the profit rate at optimum when firm $j$ maximizes the profit rate and by $\Pi_j^*$ the value of the profit rate at optimum when the firm maximizes the pure profit. As $\pi_j^*$ is obviously the maximum maximorum of the profit rate (it has been found following a maximization procedure), $\pi_j^* \geq \Pi_j^*$. Hence, the firms which are pure-profit-maximizing are less attractive for the financial shareholders (sleeping-partner shareholders are less concerned); their shares will be sold up, their value will decrease and they can be bought up more easily by raiders. Nevertheless, the reverse proposition is not true. Denote by $V_j^\Pi$ the pure profit at optimum when firm $j$ is pure-profit-maximizing and by $V_j^\pi$ the pure profit at optimum when firm $j$ is profit-rate-maximizing; $V_j^\Pi$ is the maximum maximorum of the pure profit, so $V_j^\Pi \geq V_j^\pi$.

29 The influence of stock options on managers’ behaviour is probably not limited to what is described here: it has been argued that managers owning stock options try to manipulate the price of shares at their own benefit (Aglietta and Reberioux 2005).

30 See also Luo (1995), Blume and Easley (2002), or Grüner (2003).
necessarily. Hence, for strategic shareholders, profit-rate-maximizing firms are financially less attractive than pure-profit-maximizing firms.

Both arguments seem symmetric but they are not actually. Financial shareholders are more interested by profit-rate-maximizing firms because these firms have a higher ROE or ROCE than if they would be EVA-maximizing. However it is not because strategic shareholders encourage the pure-profit-maximizing firms to maximize their EVA that they select them following the criteria of EVA: strategic shareholders do not select firms only because their EVA is higher. For strategic shareholders the first criterion is not profitability (nor EVA nor ROE-ROCE); it is different: having the vertical control on a supplier or on a customer, having a horizontal control on a potential competitor, controlling a firm of national importance, etc. No natural selection issues from strategic-shareholders’ behaviour.

4. Conclusion

Remaining inside the neoclassical corpus, this paper has considered two types of firms: those maximizing their pure or economic profit, as assumed classically in the microeconomic theory, and those maximizing their profit rate.

In the first part of the paper the behaviour of both types of firms has been compared by respect to the level of the output and to the price. In the most probable case where company’s return on employed capital is higher or equal to the weighted average cost of capital, required or expected, the output (and the input consumption) of profit-rate-maximizing firms is lower than (or equal to) those of pure-profit-maximizing firms; the price of output evolves in the opposite way. The demonstration is valid for monopoly (the price is always higher) and for perfect competition; in perfect competition with fixed coefficient of capital, the output price loses any role in the equilibrium (what implies no coordination between firms). This has been applied to the case where the capital is the total capital engaged (EVA versus ROCE) or where the capital is the equity (EVA versus ROE). These results cannot be transposed, \textit{mutatis mutandis}, to all stakeholders different to shareholders: if it is possible to compute a rent, by transposing the derivation of the economic profit, computing a ratio by transposing the derivation of the profit ratio is nonsense or trivial.\footnote{On how the stakeholders are taken into account and the rent is computed, see Milgrom Roberts (1992).}

Part 2 has explored how shareholders’ behaviour may influence companies’ objective with a microeconomic point-of-view. A typology of firms’ behaviours has been proposed (leaving aside the questions of corporate governance or agency theory).

In the first case, shareholders try to maintain fixed their control rate on firms, having a “strategic behaviour”: they maximize their own net income (taking into account the opportunity cost of the equity capital) which includes companies’ distributed profit. Hence companies maximize their economic profit.

In the second case, shareholders have a financial behaviour and control the composition of their portfolio, allocating freely their equity capital between firms: they maximize the return on their equity capital. Hence companies are encouraged to maximize their profit rate and employ fewer factors, as labour: this corresponds to shareholders’ worse behaviour. The combination of these two behaviours has been considered.

The possible switching from a strategic behaviour (economic-profit maximization) to a financial behaviour (profit-rate maximization) may explain why companies can lay off workers while they are largely profitable: their shareholding may have switched from one behaviour to the other; applied studies could verify this. The natural selection is in favour of
profit-rate-maximizing firms owned by financial shareholders. We do not say that this switching explains all the recent behaviour of large firms, the paradox presented in introduction, but that it could at least explain a large part of it. This should be tested. Another job.

A third case has been considered also. Shareholders may have a sleeping-partner behaviour, letting their equity invested in the firm for a long time, without subscribing to any new issue of shares. They maximize the return on their equity and so the companies maximize their ROE (or ROCE if the cost of debts and the debt to capital ratio are fixed). However, because of their inertia, they have a small influence on the firm, only ex ante and not ex post. This allows showing that stock options and other forms of profit-sharing may conduct EVA maximizing firms to maximize partially the ROE or ROCE.

5. Annex

5.1. The classical alternative theories to pure profit maximization

Many authors have proposed a certain number of theories, alternative to profit maximization as firms' behaviour, coming out of the neoclassical corpus. Leaving aside the well-known discussion on the plausibility of the marginalist behaviour of companies: see for example Hall and Hitch (1939), Lester (1946), Machlup (1946) (this discussion turns around the idea of existence of routines inside the firm, mechanically applied, far from the idea of maximization as a goal, but which induce a global behaviour of the industry as if firms were maximizing the profit by following a natural selection argument (Alchian, 1950)), there is: the theory of separation between ownership and control itself (Berle and Means 1932) (see also Pitelis (1986) or Putterman and Kroszner (1996, p. 25)), the managerial theory with Baumol's maximization of firm's size or income (1959); see also Klemm (2004) who considers an oligopoly where at least one firm maximizes or minimizes its output, Marris' maximization of growth (1964) and Williamson's maximization of manager's utility (1964); see also Holmstrom and Tirole (1988 pp. 104-5), the behaviourist theory of the firm (Cyert and March 1963), the theory of contracts and incentives (Hart 1983).

Some pricing rules may conduct to non-profit-maximization: maximization of added-value for labour managed firms (Steinherr 1975; Sertel 1982; Miyazaki and Neary 1983), maximization of total surplus under a constraint of nonnegative profits for natural monopolies (Ramsey-Boiteux pricing; see Ramsey (1927, 1928) and Boiteux (1956)); for a non-profit organization, like a state-owned firm, a mutual benefit society, a hospital or a charity organization, the objective could be simply to equilibrate the accounts or to maximize the consumer's surplus (Lynk and Neumann 1998) but sometimes it can switch to a for-profit status (Picone et alii 2002); see Rose-Ackerman (1996) for the link between altruism and non-profit organizations. One may quote also the maximization of the mean profit by unit of output. In some cases, some of these behaviours can be combined for the same company: see for example Zabojnik (1998) for a firm which maximizes both profit and sales or Denef and Masson (2002) for a hospital which maximizes both profit and output.

Probably there are other theories that can determine possible firms' behaviours: the theory of organizations (Cf. Simon's distinction between the Firm Theory (F-theory) and the Organization Theory (O-theory) (Simon, 1952-53; Imai and Itami, 1984)), the theory of bounded rationality (Simon 1962), the transaction-costs theory (Coase 1937; Williamson 1975; Milgrom and Roberts 1992), and finally the melioration theory (Mainwaring 1997).
Melioration is the observed behaviour of individuals such that they have difficulties to maximize when the choice depends on the way similar choices have been distributed in the past.

5.2. The function of capital: discussion

We have assumed that $K'(Q) > 0$. However one could discuss to know if $K$ is a known monotonous function of $Q$. First, it must be noticed that the following discussion concerns both profit-rate and pure-profit hypotheses, and not only the profit-rate one: even if in microeconomics the opportunity cost of capital is usually not taken into account to determine the pure profit, it should be beyond a vague idea of “cost”... The output $Q$ depends ex ante on the scale of production but the scale of production depends itself on the amount of net asset (that is, physical capital) bought. On the other hand, ex post, the (financial) capital used in the production is not directly linked to the amount of physical capital used, while if the amount of financial capital is measured by its market value, it depends on the profitability wanted by shareholders.

The problem is even recursive in the sense that the amount of capital measured by the value of the firm is equal to the actualized sum of profits, while the profit depends on the amount of assets installed, that is, on the quantity of capital... In other words, looking at the present output, the link with the present amount of engaged capital is not so clear. If the market thinks that firm's future is bad (respectively, good), the value of firm's share will go down (respectively, up), then the value of the firm will go down (respectively, up); for the same $Q$, $K(Q)$ can be high or low: monotony is not guaranteed, but for each level of $Q$ is associated a certain level of $\Pi$, so a certain level of the actualized sum of future profits and a certain level of $K$, all things being equal by elsewhere. Notice that if $K$ is not a function of $Q$, no microeconomic reasoning is possible because even the pure profit becomes impossible to maximize as a function of $Q$, and not only the profit rate.

Moreover, one could discuss the exact value of $K'(Q)$.

- If $K'(Q) = 0$ then $\pi(Q^*) = \pi^*$, then both equilibrium (profit rate and economic profit) coincide. Is it plausible to have $K'(Q) = 0$ that is $K(Q) = k$? One could think that it is the case of fixed costs. This is false because the level of fixed costs depends on the capacity of production; with fixed costs the capital function resemble to stairs: a level $K_0$ is valid for $Q \in [0, Q_0]$ then a level $K_1$ is valid for $Q \in [Q_0, Q]$ with $K_1 > K_0$, etc. Actually, $K(Q) = k$ is the case of a production with there are no fixed costs at all but only circulating capital advanced to buy intermediary commodities and salaries, e.g., for the first month: the same capital could serve to produce during one month or years. Realistic only for individual productions.

- If $K'(Q) < 0$ all results of Theorem 1 are reversed. It is not realistic even if there are increasing returns. Increasing returns do not affect $K'(Q)$ which remains non-negative (to double the production, one must increase the capital) but only $K''(Q)$ with $K''(Q) < 0$ (to double the production, the capital is less than doubled).

It should be notices that Katzner’s function of capital is a little more sophisticated than ours as it includes (Katzner 2006, pp. 542 and 554), in addition to the output, the inputs, the output prices and the input prices (for the interested reader, he uses other notations: $x$ for the outputs, $y$ for the inputs, $p$ for the output prices and $r$ for the input prices). However, this can be justified. First the quantities of factors are linked to the outputs by the production
function (as in Katzner’s book, p. 551): if the production function is included in the function of capital, the quantities of factors may be removed from this function. Second, Katzner is in the context of a general equilibrium (hence, in perfect competition) while we are in partial equilibrium, in imperfect as well as in perfect competition: as the factor prices are given, we are able to omit them in the capital function.

6. Bibliographical references


