RISK, POLICY, AND COEXISTENCE EQUILIBRIUM

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Background: A new treatment is presented for the efficient organization problems. Hypothesis: The conflicting theories coexist. A testable assumption is that the correlations, rather than expectations, are rational and not limited within a unit root. Methods: The rational correlation of related variables is measurable and computable by dynamic quadratic regression. Findings: The policy dose is converted into the non-zero coexistence equilibrium, which is the boundary value of organizations. Conclusion: Evidence supports the non-zero coexistence hypothesis with the conversion probability between centralization and decentralization and between output and input over time. (JEL A10, B10, C10, D10, E10)

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1.1 INTRODUCTION

Two players bargain and resort to the third player over time. Each player intends to maximize the expected gain of wellbeing, and choose the best response of policy. The third player sets the transparent regulatory policy and reduces the cheating behaviors. Consider the age-old problem. Why do some groups succeed in promoting cooperation while others suffer from conflicts?. We find the coexistence of conflicting theories:

THE NULL CONJECTURE 1a is that the capability for uncertain retaliation is more credible and more useful than the ability to resist an attack. No statistical test has been done (Schelling, 2006, page 937).

In one-time games, an organization can strengthen its position by overtly worsening its own options. Uncertain retaliation in the nuclear arms race is more credible than certain retaliation.

THE ALTERNATIVE CONJECTURE 1b is that in repeated games, denial of the capability for uncertain retaliation is more credible and more useful than the ability to resist an attack. Uncertain incentives tend to reduce cooperation. In multiple equilibria, players are risk-averse and learn to move out of the weak position into the strong equilibrium.

The first-order best solution is the sign-changing critical point. Players change their behavioral correlation. Competing risks and conflicts appear. No testable assumption, however, was made about finding the policy dose of promising players, and fits the data about the accelerated failure time to improve identification. Thus, no solid theoretical foundation is proposed; the hazard function is estimated by linear programming under the assumption of independence distribution (Honore, et al. 2006). A question arises. How can we identify the proper policy dose, avoid self -destruction and retaliation, and promote our wellbeing?. Few works have transformed the policy responses into the coexistence equilibrium in a continuous sense.

The thesis of this essay is that the coexistence equilibrium is the group's exact stable solution; under this condition, action may be taken by one of individuals without the consent of the other and can affect the wellbeing of the other one. The uncertainty has a positive and a negative effect on organization forms.

In the following, in Section 1.2, a simple model can capture the coexistence of conflicting theories; Section 1.3 presents a story of theories; Section 2 shows the model and method about the cooperative and competitive risks. The readers who are not interested in mathematical computing can skip the proof in Section 2. Section 3 concludes with remarks.

1.2 THOERY

Our core conjecture is that the coexistence equilibrium is a unique, stable, and non-zero minmax solution and is more useful than the capability for uncertain retaliation. In equilibrium, errors, conflict and retaliation vanish. A solution is a definite amount which is determined by the finite correlation of related variables. Our testable assumption is that at least a sequence of observations of related variables is serially correlated. The equilibrium is a dense, constant, or quasi-stationary fixed point, is the resource- and energy- balance of related variables, and is an expected boundary value. The equilibrium organization is measurable at a stable location, and has a dense probability distribution for a finite duration time.

The equilibrium policy is the best policy dose and measured in units of time and distance, and is a rule of law, explaining the reality and the evolution of realities. Such a policy dose is applicable to the proper drug dose, the inventory duration, the finite life-time of patents, the moratorium of take-over, life cycles of earning, saving, products, and business, the transportation plan in infinite horizon, as well as other organization forms.

The correlation is a rational number between related variables, input and output, and can transform from a monetary price unit into a real price unit. The capability is potential causal powers and is measured by rational correlation. The performance criterion of wellbeing is indicated by the probability of convergence and conversion, within a finite time, from input to output, or from a correlation into the equilibrium of related variable. The convergence probability is an inverse of uncertainty, variance, or risk of outcomes.

In methodology, the curve of equilibrium is estimated by the dynamic quadratic regression, where the competing risks are reduced to zero. We solve the correlation in an adjoint evolution equation and transform the mathematic proof into the dynamic statistic test. Every variable can be treated as endogenous and exogenous variables in a simultaneous equation system.

In simulations, equilibrium satisfies almost all constraints and avoids the non-invertibility of a negative eigenvalue in linear equations. Risk and oscillation are found in the time-varying correlation of related variables, but not predicted in the linear models. Mutual causality implies that the related players in organizations translate information through a foreword and backward communication rather than in a forward straight line. Below or beyond the equilibrium, the correlation response of policy uncertainty switches its sign from a positive or cooperative into a negative or competitive response over time.

1.3 A STORY

In the imperfect markets, players' correlation is sign-changing over time. Players lend their saving to other players, who are managers. The players use the interest rate to control the managers' performance, ensuring the minimum stock return. As the age and the firm's size grow, the firm survives and reverts to a stable relation between investment and saving over time. As uncertainty of relationship declines; the equilibrium is the expected mean value, while the sample average does not allow for effects of early experience.

In macroeconomics, players are consumers and investors, and organize the firms and the governments, which subsidizes education and research. The equilibrium time is a finite time of patent rights and a trade-off between the invention diffusion and monopolistic incentives. Nevertheless, the natural rate is less useful than the equilibrium, where players can earn higher than the minimum equilibrium wage by law, and may suffer from the minimum unemployment rate which is lower than the natural rate of unemployment. Similarly, the natural rate of interest is less useful than the equilibrium. The equilibrium real rate of interest is correlated with the real money balance and money growth, is equal to the actual minimum or maximum rate, and is sustainable by the central bank's policy.

In simulation, linear models yield the first-order best solution, such as the zero inflation rate, zero interest rate, and zero marginal income tax rates, which is the infeasible solution. If the sign of correlation coefficients is restricted to be positive, and if import oil prices are rising, the central bank could accommodate such external shocks, expanding the money supply to increase output. Similarly, the government can expand the unemployment insurance benefit generously. Beyond the equilibrium, after shocks vanish, such excessive expansionary policy has negative effects and leads to the hysteresis of high unemployment rate and high inflation rate. Business cycles need not be intervened, if the correlation automatically switches around the equilibrium. The Great Depression and crises, however, need be intervened by the equilibrium policy.

For households and culture, equilibrium is a mating stable relation. As the demand (or time preference) and the supply (or technology) changes, the labor supply curve becomes backwards bending. The correlation between fertility and labor force participation is positive before 2001 and negative after 2001 in Europe. The household income is stabilized by the complementary or substitutable (un)employment rates between wife and husband.

Under the group decision, players have belief and incentives, take and diversify the risk, and coordinate specialization and division of labor. Beyond (or below) the equilibrium, players sell (or buy) across spaces and over time, use the spot (or forward) contract, or work in the domestic (or foreign) and detailed (or wholesale) related markets. Furthermore, the correlation shows a sign-changing relationship between sales and investment, sustaining their equilibrium ratio. For example, inventories are part of investment. When sales increase, the planned and voluntary inventory increases; when sales decrease, the unplanned and involuntary inventory soars. Thus, the firms adapt and reorganize within a short time, and adopt the just-in-time delivery of products and services to customers and markets.

2 MODEL AND METHOD

PROPOSITION 1: *In the concave or inverted U shape of welfare functions, over the finite and infinite horizon, beyond (or below) the equilibrium confidence region, the response of equilibrium policy has at most one sign change, and is a trade-off between the positive and negative correlation. Excessive regulatory intervention has crowding-out effects and reduces productivity.*

Production is a roundabout procedure. In static analysis, two players bargain and generate the first best response of output, say, $\theta_1 = Y$ = 4=2+2, which is then shared or divided by two, θ_1/2=Y/2, and multiplied by the output price, *P.* The equilibrium output is equal to each player's income:

(2.1a) $\theta_1 / 2\theta_2 = (Y/2)P=(4)/2(1)=2$

where θ is a correlation response. Y is output of, say, energy. P is the price.

In dynamic spatio -temporal activities, players consider an additional dimension, $0<t\leq n$, such as a time, a space, or the number of workers (or cells). Coexistence equilibrium is a sustainable output and input over time, and independent of positive or negative correlations, θ.

(2.1b) $-\theta_1/2(-\theta_2)$=YP/2=Y(t)(1/r(t))/2 for $0<t<n$ *as* $n\to\infty$

where YP is the value of the firm, and is the discounted present value of the output over time. The price *P=1/r* is the discount present value of input; r is the input, such as policy doses, foods, drugs, or the discount interest rate.

DEFINITION 1: The *correlation* θ is a rational *coefficient* of related variables, output *Y(t)*, and input time *t.*

$(2.2)\quad \theta(t)=Y(t)/t \qquad \text{for } 0<t\leq n$

where correlations, $\theta=\lambda(\theta_0,\theta_1,\theta_2,\theta_3,\theta_4)$, are called a Lagrangian multiplier and denotes the opportunity cost and price of resource constraints. The correlation is positive (or negative), denoting an accelerator (or a decelerator), cooperation (or competition), and friendly (or retaliating) effects.

Our objective is to estimate the non-zero equilibrium in the least squares space:

$$(2.3)\quad H(x(t))=\max_{x\subset\Omega}\ \min_{t\Omega}\sum_{t=1}^{n}(p(x(t))-F_t(x(t)))^2=0 \qquad \text{for } 0<t\leq n \text{ and as } n\to\infty$$

subject to the evolution equation:

$$(2.4)\quad x(t)=F(x,t\text{-}1)=F_t(x(t)) \qquad \text{for } t>0$$

where the time t is a background function. In (2.3), *H*, *p*, and *F* are the continuous thrice-differentiable functions. Ω is a compact convex continuous domain of organization. In (2.4), the evolution function is a stable relationship of output growth, *x*, over time.

The procedure of computing the equilibrium is based on dynamic quadratic regression as follows:

STEP 1:Initialize the sample of observations (x(t), r(t)) for 0<t$\leq$n.

STEP 2:Estimate the second-order evolution equation:

$$(2.5)\qquad dx/dt\approx x(t)-x(t-1)$$

$$= F_t(x(t),r(t))$$

$$= \theta_0 + \theta_1 x(t-1) + \theta_2 x^2(t-1) + \theta_3 r(t-1) + \theta_4 r^2(t-1) + v(t)$$

where v is remainder errors and vanishes in equilibrium.

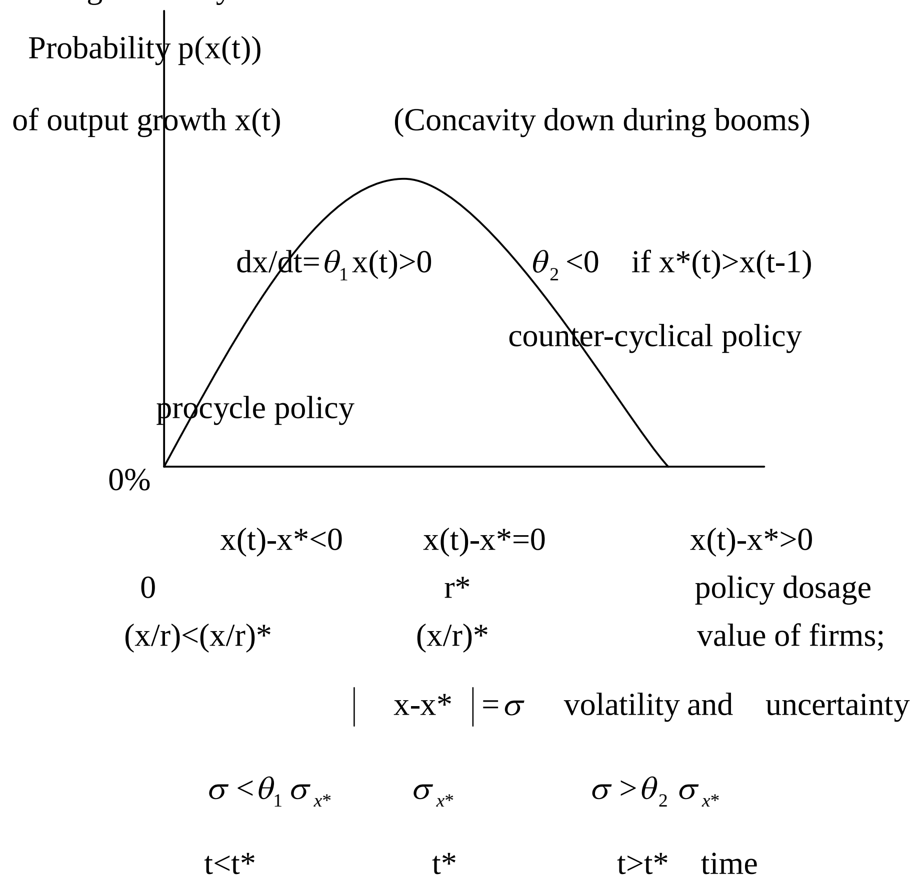
STEP 3:Estimate the equilibrium,

$$(2.6) \quad x(t)-x(t-1) \quad = \theta_2 (x(t-1)-x^*)^2 \quad + \theta_4 (r(t-1)-r^*)^2$$

$$=0 \qquad \text{if } x(t)=x(t-1)=x^* \text{ and } r(t)=r(t-1)=r^* \text{ for } t>0$$

We test the null hypothesis: $H_0 : \theta = \theta(x)=0$ versus the alternative: $H_1 : \theta = \theta(x) \neq 0$ in the same least squares space. Figure 1 shows the concave or inverted U shaped domain of utility. In (2.5), the equilibrium output growth is $x^*=x=x(t)=x(t-1) =- \theta_1/2\theta_2$; the equilibrium input is $r^*=- \theta_3/2\theta_4$.

Figure 1. Payoff Outcomes and Time Period



short-run long-run optimal stopping time

F(x,r,t) Organization function

DEFINITION 2: The *second-order partial derivative* is defined as the variance,

$$(2.7) \quad dx(t)/dt = \theta_0 + \theta_1 \frac{\partial F_t(x(t))}{\partial x(t)} + \theta_2 \frac{\partial^2 F_t(x(t))}{\partial x^2(t)}$$

$$= \theta\,(x(t)-x^*)^2$$

$$= \theta\,\sigma_x^2$$

where σ_x^2 denotes the uncertainty, volatility, ambiguity, velocity, or the variance of outcomes x*(t)*.

DEFINITION 3: The *convergence probability,* $0<p(x(t))\leq 1$, is the probability of the observations converging towards the equilibrium x* for all players.

$$(2.8) \quad p(x(t))=exp(dx(t)/dt)=exp(d\,F_t\,(x(t))/\partial\,t^2) \qquad for\ 0<t<n\ and\ as\ n\to\infty$$

As $x(t)\to x^*$, $p(x^*)\to 1$ for t>0. The symbol $\to$ denotes the convergence. $0<p(x)\leq 1$ is the *probability of convergence*, implying usefulness, acceptability, controllability, or conversion from input to output or from ideas to patents.

DEFINITION 4:The equilibrium, *x(t)=x(t-1)=x** for *t>0*, is a unique stable solution between related variable *x(t)*, and estimated by correlation coefficients θ over time. The stable relation of variable *x(t)* and correlation θ exists such that

$$(2.9) \quad x^* = -\theta_1/2\theta_2 \ \text{ for } t>0 \text{ if } x^*(t)=\frac{-\theta_1 \pm \sqrt{\theta_1^2 - 4\theta_0\theta_2}}{2\theta_2} \ \text{ and } \ \theta_1^2 - 4\theta_0\,\theta_2 = 0.$$

An asterisk denotes the equilibrium.

PROPOSITION 2: *Compensation for loss and injury is equal to the deviation in disequilibrium. Oscillation, explosion or extinction may recur due to the persistent positive or negative coefficient. Thus, if the linear response,* $0<\theta<\infty$, *is sign-restricted or limited within one unit circle,* $|\,dY(t)/dP(t)\,| = \theta < 1$, *the equilibrium blows up to become zero,* $x(t)=x^* \rightarrow 0$ *and* $Y(t)=Y^* \rightarrow Y(0)$ *as* $t \rightarrow \infty$.

The equilibrium policy dose is the maximum likelihood estimator, and is the expected value in the discounted present value of utility and the cumulative wealth function at time t. When the interest rate rises, saving, investment and output increases. As interest rates r(t) remains below the equilibrium and have positive effects, the output expands. The linear supply curve is positively slopping

(2.10a) $x(t)=\theta_4\ r(t)+\theta_2(x(t-1)-x^*)$

$\theta_4 r(t)=\alpha>0$, and $\theta_2=\beta>0$ if $r(t)<r^*$ and $x(t-1)<x^*$

Beyond the equilibrium, as the interest cost of policy input r(t) rises, excess return declines and has negative effects on output growth. The linear demand curve is negatively– slopping:

(2.10b) $x(t)=-\theta_3 r(t)+\theta_1(x(t-1)-x^*)$ $-\theta_3<0$

$-\theta_3<0$ and $\theta_1>0$ if $r(t)>r^*$ and $x(t-1)<x^*$

where $d\log Y/dt=x(t)$ is output growth or stock return. r is the riskless interest rate.

REMARK 1: For the firms, as in (2.10a), the t-th firm's excess return is positive, $x(t)-r(t)>0$; the market excess return is positive, $x^*(t)-r^*(t)>0$. In (2.10b), the organization with a persistently negative return, x(t)<0, tends to die out. The equilibrium is injury compensation, where workers' injury or loss is compensated by the social insurance and private precautionary saving over the injury period, $0<t\leq n$. The injury has the negative effect on output:

(2.11) $\sum_{t=0}^{n}$ (Y(t)-Y*(t))=*Y(0)exp* $\sum_{t=1}^{n}$ *(x(t)-x*(t))*

where the equilibrium (*Y*, x**) is the expected output and output growth. The output loss occurs due to injury, Y(t)-*Y*<0*. *n* is the terminal time of injury.

PROPOSITION 3: *The competing risk or variances vanish at equilibrium, when a negative correlation is transformed into a positive one. Equilibrium resolves the problem of risk,* $(\theta\,(x(t)-x^*))^2=(-\lambda)^2$*, and the invertible negative eigenvalue in linear models.*

The equilibrium reduces the uncertainty, and is a trade-off between risk and excess return. If the ratio of excess return to risk is infinite; its inverse is finite and solvable:

(2.12) $\lim_{n\to\infty}\sum_{t=1}^{n}\dfrac{\theta\lambda(t)}{|\lambda(t)|^2}=\dfrac{x(t)-x^*}{\sigma_x^2}=\infty$, and $\lim_{n\to\infty}\sum_{t=1}^{n}\dfrac{|\lambda(t)|^2}{\theta\lambda(t)}<\infty$

if $(x(t)-x^*)=\lambda$

where the eigenvalue λ is the risk premium. We use the rational correlation, $(\theta)^2=(-1)^2,=(-\lambda)^2$, and solve the invertible negative eigenvalue and irrational complex number, $\sqrt{-1}$. Suppose the operation of logarithm is used to transform from the division into a minus relationship, log(x/r)=log x – log r, and from a multiplication into a summation , (log YP)=(log Y+logP). We use the equilibrium to transform the negative correlation (x*=- $\theta_1/2(-\theta_2)$ into a positive equilibrium, (x*=$\theta_1/2(\theta_2)$)).

If the correlation is unity, the equilibrium is an identity condition:

$$(2.13)\quad 1=\theta_2\frac{C(t)}{Y(t)}+\theta_4\frac{I(t)}{Y(t)}+\theta_6\frac{G(t)}{Y(t)}\qquad \text{for}\quad \theta=\theta_2=\theta_4=\theta_6=1$$

where input *Y* equals output, and is the sum of consumption *C*, investment *I*, and government spending *G*.

We use the dynamic quadratic regression, and estimate the equilibrium policy and convergence probability:

$$(2.14)\quad p(Y^*)=exp(\frac{d}{dt}\frac{C(t)}{Y(t)})$$

$$=exp\,(\theta_2(\frac{C(t)}{Y(t)}-\frac{C^*}{Y^*})^2+\theta_4(\frac{I(t)}{Y(t)}-\frac{I^*}{Y^*})^2+\theta_6(\frac{G(t)}{Y(t)}-\frac{T^*}{Y^*})^2)\rightarrow 1$$

if C(t)=C, I(t)=I*, G(t)=G*, Y(t)=Y**

If government spending G increases, then investment I, consumption C, and output Y rise. They are positively correlated, $\theta>0$, and revert to equilibrium. Beyond the equilibrium, the excessive government spending tends to crowd out investment or consumption, and reduces output growth. They become negatively related, $\theta<0$. Thus, the best policy dose is the equilibrium policy.

(2.14) is a variance decomposition; in equilibrium, the competing three risks vanish:

$$(2.15)\quad p(Y^*)=exp(\theta_2\,\sigma^2_{c/y}+\theta_4\,\sigma^2_{I/Y}+\theta_6\,\sigma^2_{T/Y})\rightarrow 1\quad as\ \ \sigma^2\rightarrow 0,\ and\ Y(t)\ \rightarrow Y^*$$

for t=t>0*

During life cycles of business or election, after delay times, budget is periodically balanced; and government spending, G, cyclically reverts to the tax revenues, T, as $G \rightarrow T$.

PROPOSITION 4:*The adjoint equation is a rule of non-neutral policy responses* $\theta \neq 0$*. In disequilibrium, the accelerator is* $\theta > 0$*, or decelerator* $\theta < 0$*, and failure time, t*.*

Consider the production function,

$$(2.16) \quad Y=\theta P$$

where Y is output; P is the input price, or the value of physical capital. I is investment. If P>1 and $\theta<1$, Y=1, the accelerator principle is

$$(2.17) \quad \Delta Y(t)= \theta I(t)= \theta \Delta P(t)$$

where, suppose Y=3P; ΔY =3I. One dollar investment, $I=\Delta P=\$1$, produces an accelerating increase of three dollars of output. Beyond the equilibrium, the correlation, θ, is a sign- changing multiplier of over- or under-investment. Similarly,

$$(2.18) \quad \Delta x(t)= \theta \Delta x(t^*)$$

where x is output growth. $\theta>0$ *(or <0)* is an accelerator or decelerator.

DEFINITION 5: An adjoint equation is a rule of adjusting the responses through error-correction:

$$(2.19) \quad \theta(t)= \theta(t-1)+ \theta(x(t)-x(t-1))$$

where the correlation is a ratio of, say, stock return to interest rates. The policy -makers (or cells) use their response estimate θ to update their beliefs about the response curve, and estimate input r and output x.

The adjoint equation shows the time-varying response coefficients:

(2.20) $\frac{d\theta(t)}{dt} = \theta(t) - \theta(t-1)$

$$= \frac{\partial H}{\partial x} = \theta \frac{\partial F_t(x(t), r(t))}{\partial x(t)} \qquad \textit{for} \qquad \theta > 0 \qquad \textit{if } x(t) < x^*$$

(2.21) $\frac{d\theta(t)}{dt} = -\frac{\partial H}{\partial x} = -\theta \frac{\partial F_t(x(t), r(t))}{\partial x(t)} \qquad \textit{for} \qquad -\theta < 0 \qquad \textit{if } x(t) > x^*$

Below or beyond the equilibrium, the response coefficient switches its sign.

PROPOSITION 5: The performance criterion is indicated by the equilibrium in an welfare-improving sequence, restoring the stable output growth:

(2.22) $Y^*(t) = Y(t) \geq Y(t-1) \geq Y(t-2), \ldots$ *as* $x^*(t) = x(t) \geq x(t-1) \geq x(t-2), \ldots$

if $r(t) = r^*$ *for* $t > 0$

where (2.22) is a welfare-improving sequence of outcomes. The equilibrium input reduces the uncertainty and volatility of output growth.

In case of no injury, the output growth x converges to the stable equilibrium, (x(t)=x*) for t>0. The life-time cycles, such as cycles of business, saving, or products, are depicted as the Makovian chain,

(2.23) $\lim_{n\to\infty} \sum_{t=1}^{n}$ p(x(t))= $\prod_{x,t,\theta\in\Omega} \theta\ x^{\theta x} = 1^{-1}\, 1^{1} \ldots = 1$

if p(x(t))=x=x*= $\theta\ x^{\theta x} = \exp(\theta\ \sigma_x^2)$

where in (2.23), due to delay effects, the observations of output periodically return to the equilibrium, and is stabilized by the time-varying correlation of policy.

The target of the policy dose is to attain the equilibrium (x*, r*, t*):

(2.24) t*=min(t, n), r*=argmin(r(t), r(n)) x*=argmax(x(t), x(n)) for $0<t\leq t^*\leq n$

where t is the observed duration until death due to, say, earning or cancer cells; r* identifies the minimum drug dosage or cause of death. x* is the maximum life-time or output growth. The convergence probability tends to convert the mass input into energy output within a speed time. The accelerated or decelerate time of failure or success is estimated by an adjoint equation:

(2.25) $\Delta x(t)=x(t)-x(t-1)$

$$= \theta_7+\theta_8 \Delta x(t-1)+\theta_9 \Delta x^2(t-1)$$

$$=\theta_9 (\Delta x(t-1) - \Delta x(t^*))^2 \rightarrow 0,$$

$$x(t)=x(t-1)=x^* \text{ as } r(t)=r^* \text{ for } t=t^*>0$$

where $t^*=-\theta_8/2\theta_9$ is the equilibrium failure or success time. $\Delta x(t)$ is the change in output growth rates. $\theta>0$ denotes the accelerated or increasingly successful response which reverts to the zero or non-zero coexistence equilibrium; and vice versa.

REMARK 2:The wage growth curve attains the maximum at age of, say, t*=45 and then declines acceleratingly:

(2.26) $F(x(t))=F(x(0))\exp(\theta tx(t))$

$$\theta>0, \quad x(t)<x^* \quad \text{if } t<t^*=45, \text{ and } \theta<0 \text{ if } t>t^*$$

where x is wage growth; t is age.

REMARK 3: In biology, humans coexist with virus and bacteria, but need delete infectious diseases. Equilibrium denotes the common solution of causes of diseases. A positive (or negative) correlation θ >0(or <0) denotes the recovery (or the infectious effect). x is the output growth, survival or death rates; r is the input or policy mass. r* is the minimum effort of input if $\theta_3<0$ and $\theta_4>0$. As in (2.10b), such a drug dose, r(t), can reduce the diseases to extinction, if r(t)>r*> θ x(t), x* $\rightarrow$ 0. Excess input has harmful effects on organization, however.

PROPOSITION 6: *The equilibrium, x*=1=1/x*, is robust to the invertible positive or negative effects, as shown in the floating exchange rate.*

According to the purchasing parity, the real exchange rate is the ratio of domestic price and the foreign price, and follows the evolution equation:

$$(2.27) \qquad dq/dt=\theta_2(q(t-1)-q^*)^2 \qquad for\ (P(t)/P^*(t))=q(t)=q^*=1\ for\ t>0$$

where the equilibrium q(t)=q(t-1)=q*=1 is robust to the positive or negative response coefficient, θ_2.

PROPOSITION 7:*The natural rate has a higher volatility and is less useful than the equilibrium unemployment rate which is supported by the minimum wage law.*

The relationship of unemployment rate, u(t), and the interest rate, r(t), is an inverse relationship of output growth and interest rates (x(t), r(t)):

$$(2.28) \qquad \Delta u(t)=\theta_0 - \theta_1(t-1)+ \theta_2 u^2(t-1)- \theta_3 r(t-1) + \theta_4{}^2(t-1)$$

$$(2.29) \qquad du/dt=\theta_2(u(t-1)-u^*)^2 + \theta_4(r(t-1)-r^*)^2$$

$$\theta_1<0, \quad \theta_3<0,\ if\ u(t)<u^*,\ r(t)<r^*$$

$\theta_2 > 0 \quad if\ u(t) \geq u^*, \quad \theta_4 > 0 \quad if\ r(t) \geq r^*$

where u is the unemployment rate. (u*, r*) is the minimum equilibrium. According to the Okun law, as output growth rate increases, the unemployment rate declines; they have a negative relationship. But below the equilibrium, the minimum unemployment rate is non-zero, serves as a discipline for sharking workers, and has a positive effect upon output growth. Thus, the equilibrium or minimum wage law encourages the unemployed students to go to school, has positive effects on income per capita, and does not significantly enhance the unemployment rate. Thus, the non-employment rate is defined as the involuntary unemployment rate for the last few weeks or for per capita income lower than the poverty level of, say, one dollar per day.

PROPOSITION 8: *Money growth is a cash flow and is not neutral. During recovery, below the equilibrium, a positive correlation appears among the output growth, stock return, the investment/capital ratio, and the cash flow; and vice versa.*

For the government, suppose B is debt or bonds. As far as the equilibrium output growth equals or exceeds the real interest rate, the ratio of debt to output will not cumulate. The necessary condition is an identity,

$$(2.30) \quad \frac{B(t)}{Y(t)} - \frac{B(t-1)}{Y(t-1)} = \frac{B(t-1)}{Y(t-1)}(x(t)-r(t)) \rightarrow 0 \qquad as\ t \rightarrow \infty$$

Mutual causality implies that an inverse relationship exists; if monetary policy is contractionary, the debt cumulates until the output growth converges to the real interest rates and decline to zero. The necessary and sufficient condition for the non-zero equilibrium growth is sustained by pro and counter-cyclical budget deficit:

$$(2.31) \quad \frac{B(t)}{Y(t)} - \frac{B(t-1)}{Y(t-1)} = \theta_2 (\mathrm{x(t)-r(t)})^2 + \theta_4 \left(\frac{G(t)-T(t)}{Y(t)} - \frac{G^*-T^*}{Y^*}\right)^2 \rightarrow 0$$

$\theta_2, \theta_4 > 0 (or < 0)$, $x(t)=r(t)=\Delta P(t)/P(t) \rightarrow 0$ for $0<t\leq n$ and as $n \rightarrow \infty$

subject to the budget constraint

(2.32) G(t)-T(t)= Δ(B(t))+ Δ(M(t)/P(t))

where budget deficit is financed by increases in public debt, B, and money supply, M.

The cumulative debt is accompanied by persistent budget deficit:

(2.33) B(t)=(1+r(t))B(t-1)+(G(t)-T(t))

The ratio of debts / output is bounded by the non-zero interested rate; but debt recidivism increases as the interest rate falls to zero.

For the firms, in (2.32) and (2.33), debt leverage and money supply are a source of cash flows and are transformed into investment and output, if the excess return is positive.

(2.34) $$\frac{I(t)}{Y(t)} = \theta(t) \frac{\Delta(M(t)/P(t))}{Y(t)}$$

(2.35) $$\theta > 0 \quad \text{if} \quad \frac{\partial}{\partial t}\frac{I(t)}{Y(t)} = (x(t)-r(t)) > \theta\,\sigma^2$$

where money supply and the value of physical capital is financial capital. M/P is real money balance. In (2.35), the necessary condition of mutual causality is that the excess return induces and is caused by debt leverage and investment over time. σ^2 is uncertainty or risk in output growth and interest rate. .

The sufficient condition is that beyond the equilibrium, excess return, investment and output growth decline:

(2.36) $$\frac{dx(t)}{dt}=\theta_2\ (x(t)-x^*)^2+\theta_4(\frac{I(t)}{Y(t)}-\frac{I^*}{Y^*})^2+\theta_6(\frac{\Delta(M(t)/P(t))}{Y(t)}-\frac{\Delta(M/P)^*}{Y^*})^2\to 0$$

$$\theta_2<0,\ \ \theta_4<0,\ \ \theta_6<0 \quad \text{as } t\to\infty$$

Beyond the equilibrium, the increase in budget deficit has negative effects, $\theta<0$, tends to enhance money supply, or raise the real interest rate, and reduce the investment and output growth. In (2.36), when the interest rate is high, cash flows and debt repayment are financed by selling the capital stocks. The performance criteria of reorganization is evaluated by the equilibrium level, ratio, and growth rate rather than by the negative or positive coefficient of excess return.

(2.37) $$\frac{\Delta(M/P)}{Y}=\frac{\Delta(M/P)}{M/P} \quad \text{if} \quad \sigma=1,$$

subject to the transaction equation is:

(2.38) $$Y(t)=\frac{M(t)}{P(t)}\sigma \quad \text{if} \quad \sigma=1, \quad (dY(t)/d(M(t)/P(t))\}<0, \text{ and } x(t)-r(t)<0$$

where σ is uncertainty and the income velocity of money.

The output growth is sustained by the money growth if the ratio of the real money balance to output remains constant or stable. The non-zero interest rate is a cost and a price for risk and uncertainty, and is an incentive for saving, and punishments against non-creditable debts. In (2.38), the output equals sales value and equals money supply multiplies by the velocity. During hyperinflation, beyond the equilibrium, excessive money growth is a cost of income uncertainty, increasing the velocity of money, inducing the accelerating interest rate to exceed the price inflation, and reducing real money balance, labor effort, income, and output.

3. CONCLUSION

The coexistence equilibrium is an exact, unique, and stable solution and the boundary value over time. In repeated games, the equilibrium policy is the ability to certainly treat and is more credible and more useful than the capability to uncertainly retaliate and resist the attack. Players learn by communication, experiments, and surveys, reduce uncertainty, avoid retaliation, and attain the unique equilibrium over time. Preliminary empirical evidences support this theory(Hsieh, 2002).

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